Parameterizing pressure transport terms in second- and third-order turbulence equations

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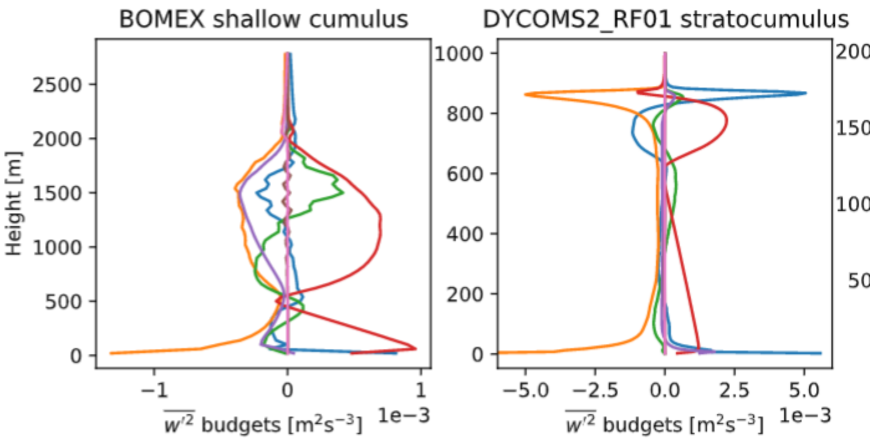
Background

- Basic wp2 equation contains a pressure term. This full pressure term can be split in two: a "pressure transport" term and a "pressure scrambling" term:

$$-\frac{1}{\rho}\left(\overline{u_i'\frac{\partial p'}{\partial x_j}}+\overline{u_j'\frac{\partial p'}{\partial x_i}}\right)=\underbrace{-\frac{1}{\rho}\left(\frac{\partial\overline{u_i'p'}}{\partial x_j}+\frac{\partial\overline{u_j'p'}}{\partial x_i}\right)}_{\text{pressure transport}}+\underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)}}_{\text{pressure scrambling}} \longrightarrow \frac{\partial\overline{w'^2}}{\partial t}=\underbrace{-\frac{\partial\overline{w'^3}}{\partial z}}_{\substack{\text{3rd-ord.}\\ \text{trans.}}}\underbrace{+\frac{2g}{\overline{\theta_v}}\overline{w'\theta_v'}}_{\text{buoy.}}\underbrace{-\frac{2}{\rho}\frac{\partial\overline{w'p'}}{\partial z}}_{\text{pres. trans.}}\underbrace{+\frac{2}{\rho}\overline{p'\frac{\partial w'}{\partial z}}}_{\text{pres. scram.}}\underbrace{-\epsilon_{ww}}_{\text{diss.}}$$

- One of these pressure terms (pressure transport) is commonly neglected in parameterizations, despite sometimes being significant in LES budgets.

Figure: wp2 budgets from SAM LES for two cases. The blue line is pressure transport.



Proposed parameterization

$$\frac{p'}{\rho} = -\frac{1}{5}\left(u'_j u'_j - \overline{u'_j u'_j}\right)$$

Lumley 1978 proposal: we expect this term to be large when 3rd- and 4th-order vertical velocity moments are large (cumulus)

$$\frac{\partial \overline{w'^2}}{\partial t} = \cdots + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho (K_{w1} + \nu_1) \frac{\partial \overline{u'_j u'_j}}{\partial z}\right) + \cdots$$

Downgradient eddy diffusion term: we expect this term to be large when gradients are large (stratocumulus)

Combining the “Lumley term” with the eddy diffusion term, in d(<w’p’>)/dz we make the replacement

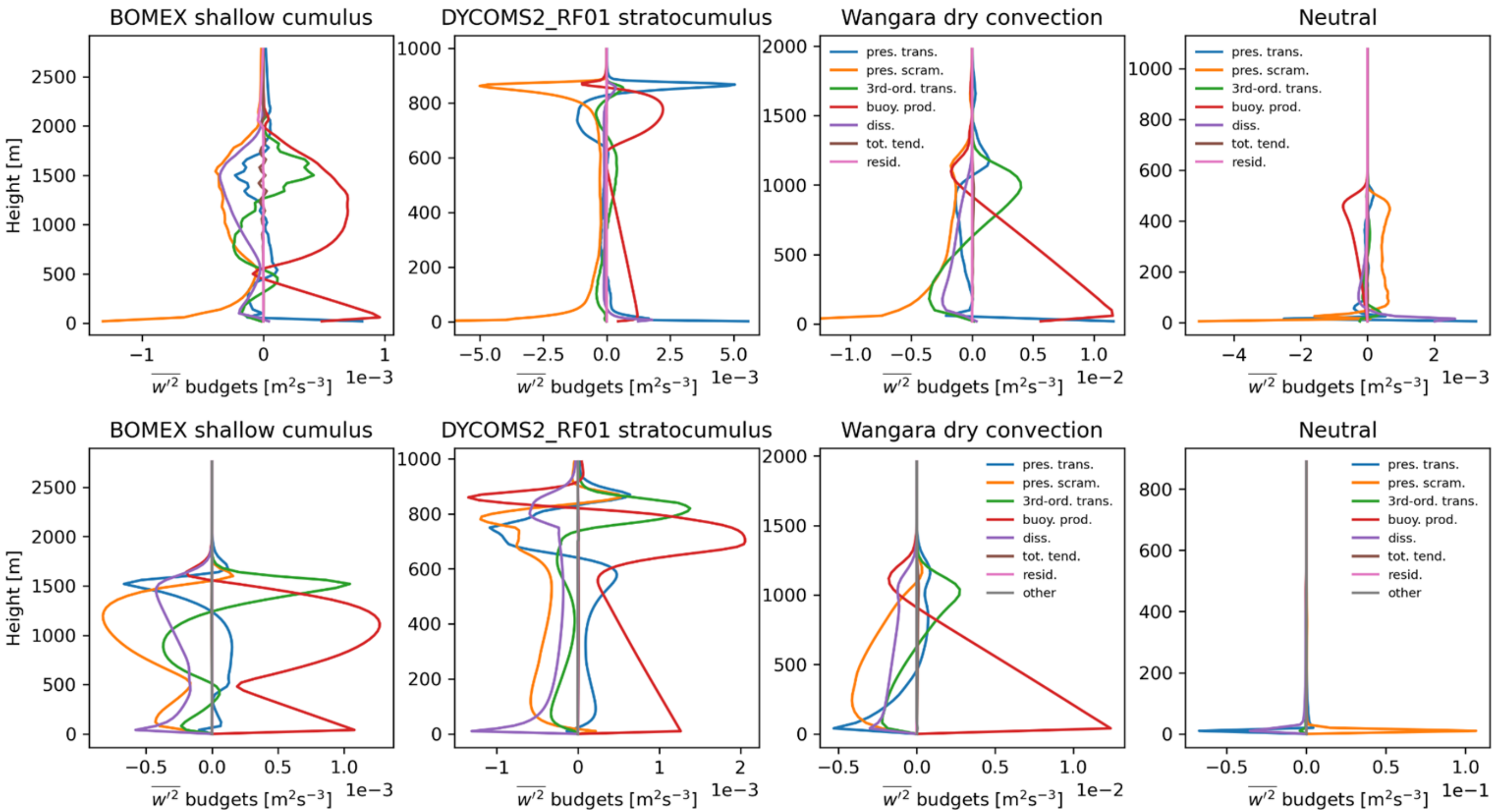
$$\overline{w'p'} \approx \underbrace{-C_{\mathrm{wp2\_lum}} \rho \overline{w' u'_j u'_j}}_{\text{Lumley}} \underbrace{- \rho (K_{w1} + \nu_1) \frac{\partial \overline{u'_j u'_j}}{\partial z}}_{\text{downgradient}}$$

where C_{wp2_lum} and nu_1 are tunable parameters and $K_{w1} = c_{K1} \overline{w'^2} \tau$

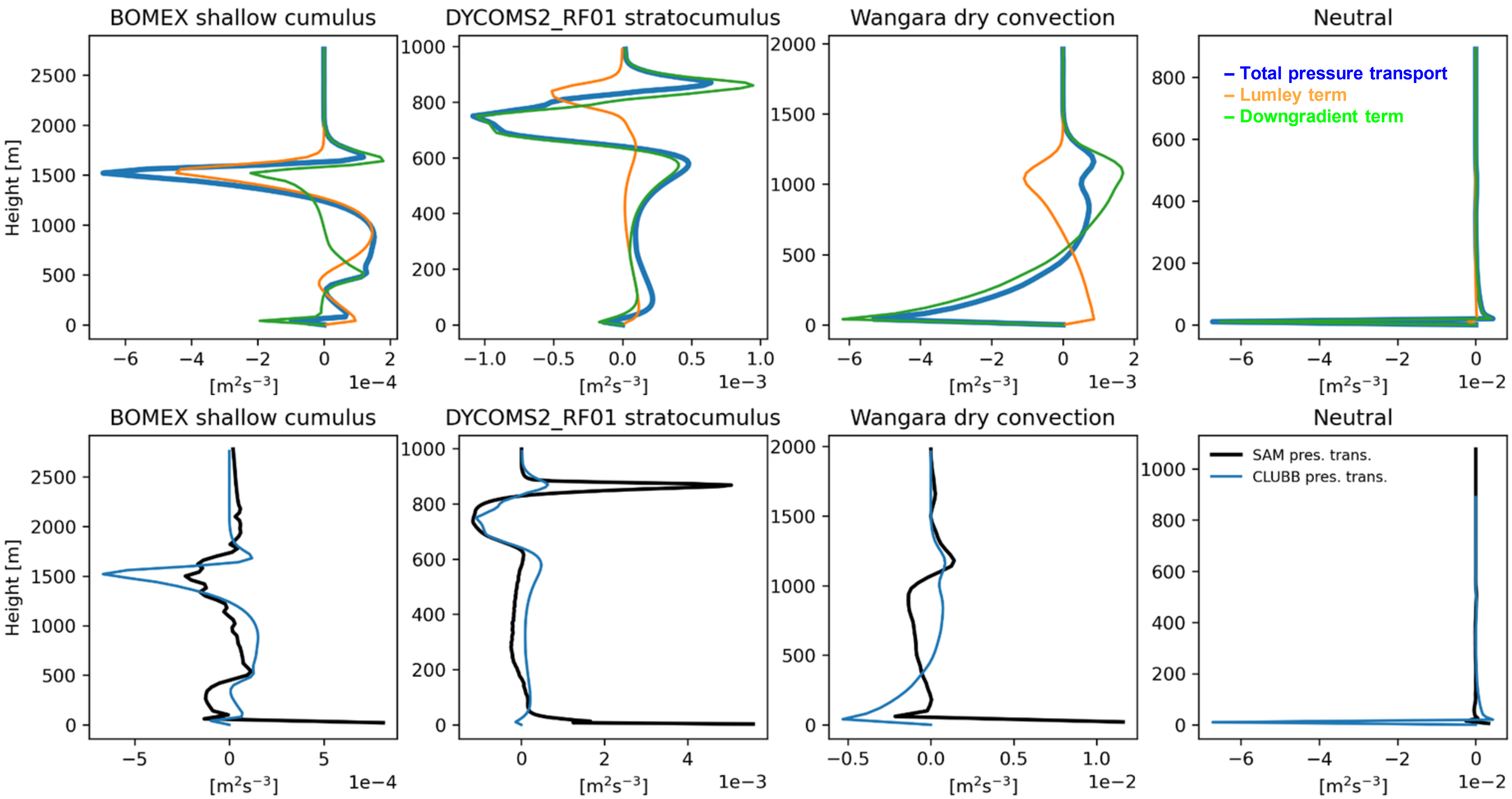
Results in SCM cases

SAM (LES)

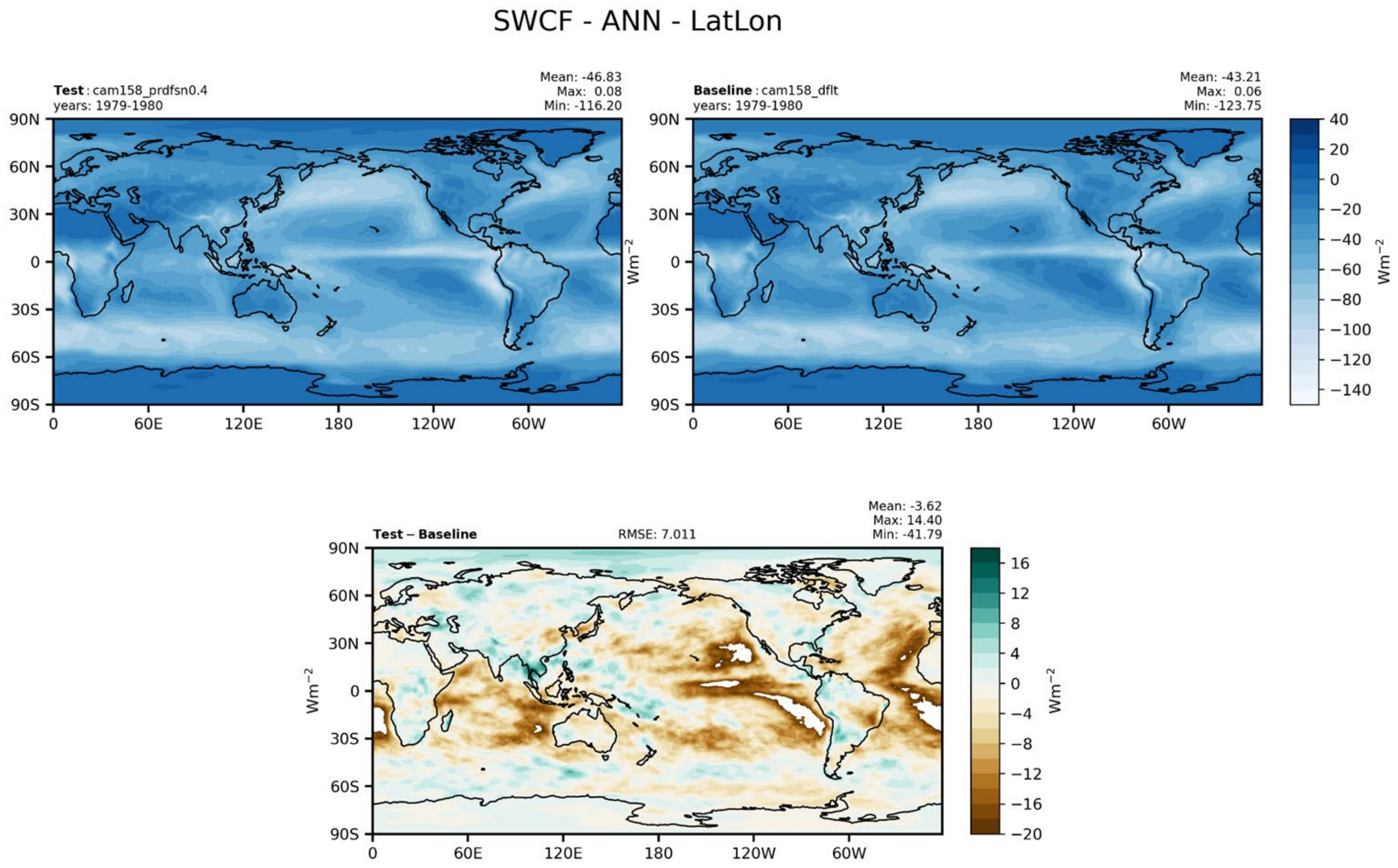
CLUBB



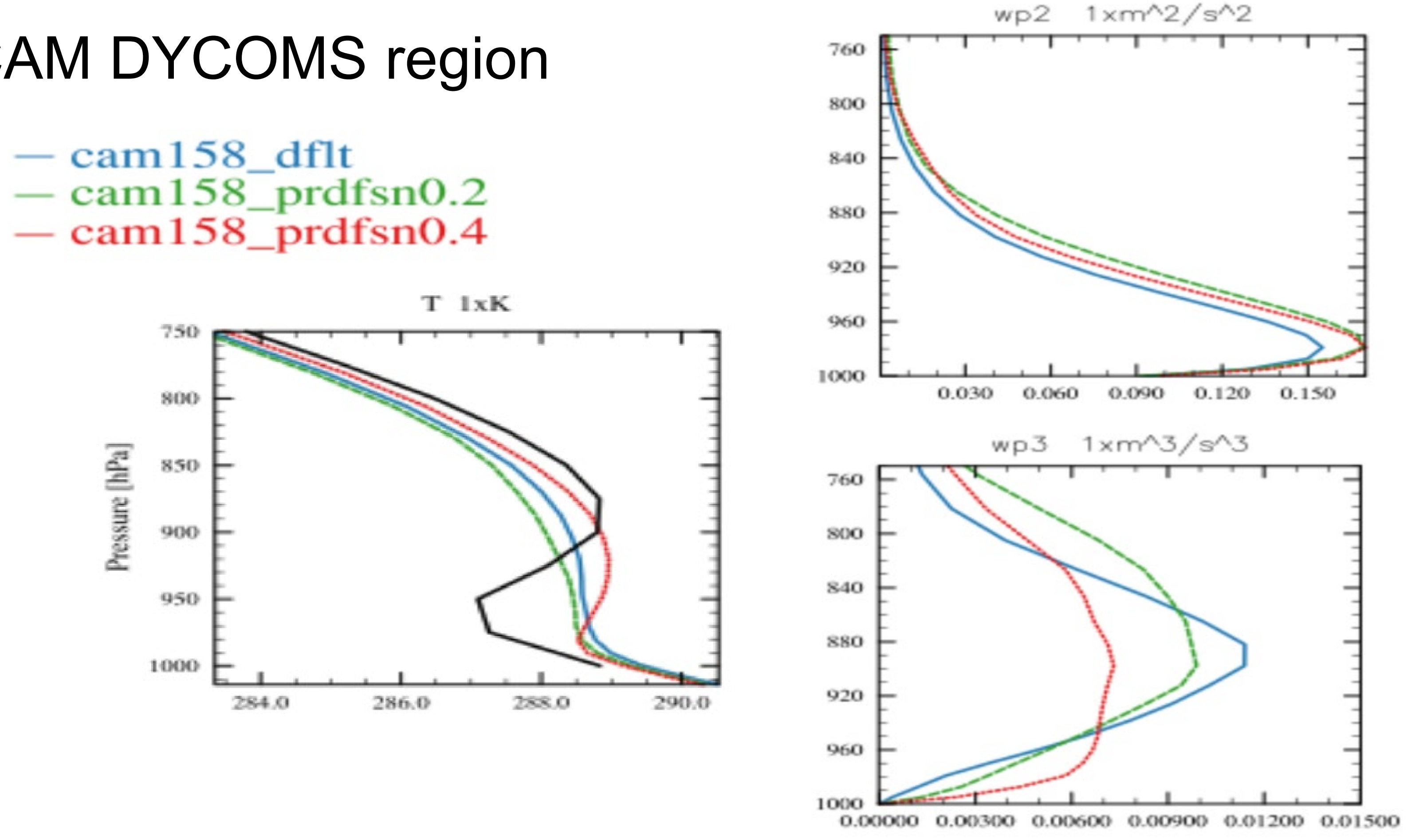
Results in SCM cases



CAM results (new)



CAM DYCOMS region



Thank you!

$$
\begin{aligned}
\frac{\partial \overline{w'^2}}{\partial t} \quad = \quad & \underbrace{-\overline{w}\frac{\partial \overline{w'^2}}{\partial z}}_{\text{mean adv.}} \; \underbrace{-\frac{1}{\rho}\frac{\partial \rho \overline{w'^3}}{\partial z}}_{\text{3rd-ord. trans.}} \; \underbrace{-2\overline{w'^2}\frac{\partial \overline{w}}{\partial z}}_{\text{shear prod.}} \underbrace{+\frac{2g}{\theta_{vs}}\overline{w'\theta'_v}}_{\text{buoy.}} \\
& \underbrace{-\frac{C_1}{\tau_{\mathrm{C1}}}\left(\overline{w'^2} - w_{\mathrm{tol}}^2\right)}_{\text{dissipation}} \underbrace{-\frac{C_4}{\tau_{\mathrm{C4}}}\left(\overline{w'^2} - \frac{2}{3}\overline{e}\right)}_{\text{return-to-isotropy}} \\
& \underbrace{-\frac{4}{3}C_{\mathrm{buoy}}\frac{g}{\theta_{vs}}\overline{w'\theta'_v} + C_{\mathrm{shr}}\left(2\overline{w'^2}\frac{\partial \overline{w}}{\partial z} - \frac{2}{3}\overline{u'w'}\frac{\partial \overline{u}}{\partial z} - \frac{2}{3}\overline{v'w'}\frac{\partial \overline{v}}{\partial z}\right)}_{\text{other pressure scrambling}} \\
& \underbrace{+C_{\mathrm{wp2\_lum}}\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{w'u'^2} + \rho\overline{w'v'^2} + \rho\overline{w'^3}\right)}_{\text{Lumley}} \\
& \underbrace{+\frac{1}{\rho}\frac{\partial}{\partial z}\left[\rho(K_{w1} + \nu_1)\frac{\partial}{\partial z}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)\right]}_{\text{downgradient}},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \overline{w'^3}}{\partial t} = & \underbrace{-\overline{w}\frac{\partial \overline{w'^3}}{\partial z}}_{\text{mean adv.}} \underbrace{-\frac{1}{\rho}\frac{\partial \rho\overline{w'^4}}{\partial z}}_{\text{4th-ord. trans.}} \underbrace{+3\frac{\overline{w'^2}}{\rho}\frac{\partial \rho\overline{w'^2}}{\partial z}}_{\text{turb. prod.}} \underbrace{-3\overline{w'^3}\frac{\partial \overline{w}}{\partial z}}_{\text{shear prod.}} \underbrace{+\frac{3g}{\theta_{vs}}\overline{w'^2\theta'_v}}_{\text{buoy.}} \\
& \underbrace{-\frac{C_8}{\tau_{C8}}\overline{w'^3}}_{\text{ret. to iso.}} \underbrace{-C_{11}\left(-3\overline{w'^3}\frac{\partial \overline{w}}{\partial z}+\frac{3g}{\theta_{vs}}\overline{w'^2\theta'_v}\right)-3C_{\mathrm{pr\_tp}}\frac{\overline{w'^2}}{\rho}\frac{\partial \rho\overline{w'^2}}{\partial z}}_{\text{other pressure scrambling}} \\
& \underbrace{+C_{\mathrm{wp3\_lum}}\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{w'^2u'^2}+\rho\overline{w'^2v'^2}+\rho\overline{w'^4}-\rho\overline{w'^2}\overline{e}\right)}_{\text{Lumley}} \\
& \underbrace{+\frac{1}{\rho}\frac{\partial}{\partial z}\left[\rho(K_{w8}+\nu_8)\frac{\partial}{\partial z}\left(\overline{w'u'^2}+\overline{w'v'^2}+\overline{w'^3}\right)\right]}_{\text{downgradient}}.
\end{aligned}
$$