Anderson Accelerated Spin Up for Earth System Models

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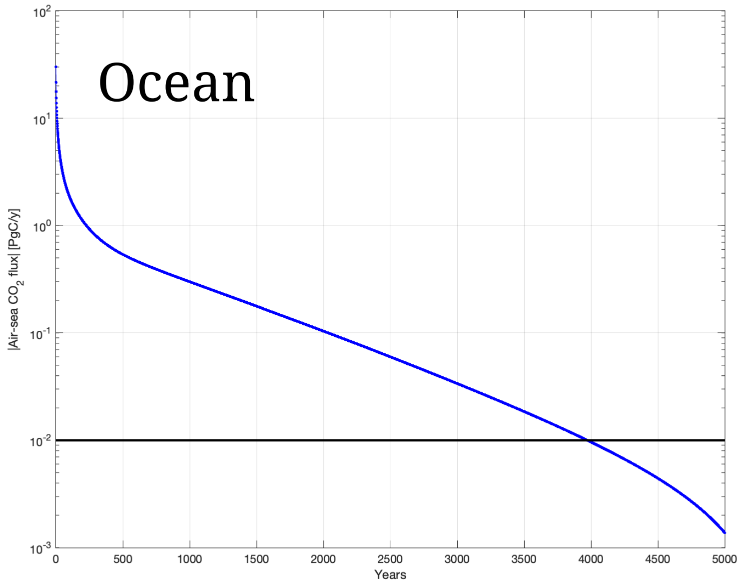
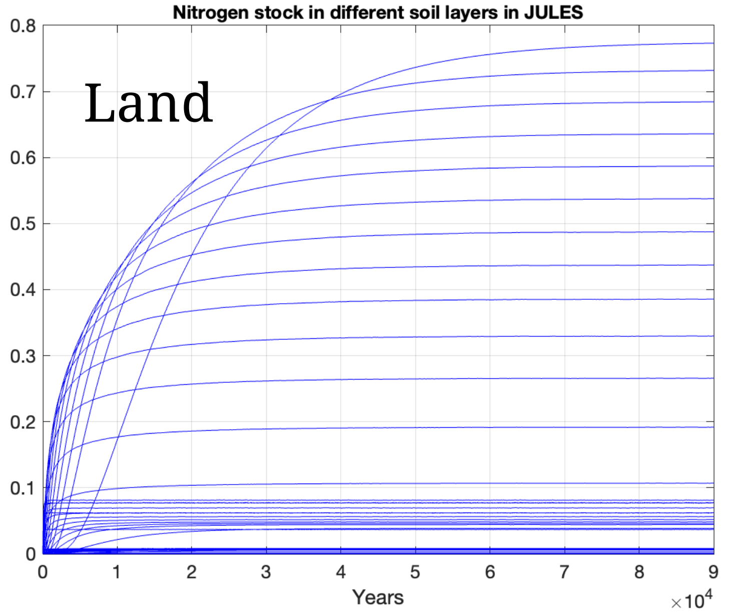
The Spin Up Problem

- Earth System Models (ESMs) need to be spun up to a steady state pre-industrial equilibrium
- Equilibration time can be extremely long due to timescales of deep ocean and terrestrial carbon cycles
- Standard approach is a Picard Iteration (Native Dynamics), is there a better way?





Samar Khatiwala. “Efficient spin-up of Earth System Models using sequence acceleration.” In: *Science Advances* 10.18 (2024).

Mathematical Background: Fixed Points

- Mathematically we are solving a fixed-point problem:

$$\mathbf{x}(T) = \mathbf{g}(\mathbf{x}(0))$$

- Native Dynamics is equivalent to a Picard Iteration:

$$\mathbf{x}_{k+1} = \mathbf{g}(\mathbf{x}_k)$$

- A Picard Iteration gives us a slowly converging sequence

$$\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots\}$$

- Our “fixed-point” is a seasonally repeating equilibrium state
- Our “fixed-point” function ***g*** is running the land or ocean model for one year
- Sequence acceleration methods try to modify this sequence to converge more quickly
  - In 1960s D. Anderson invented *Anderson Acceleration (AA)*
  - AA uses past iterates to form a best next guess at the fixed point

Donald Anderson. “Iterative Procedures for Nonlinear Integral Equations.” In: *J. ACM* 12.4 (1965), pp. 547-560.

Mathematical Background: Anderson

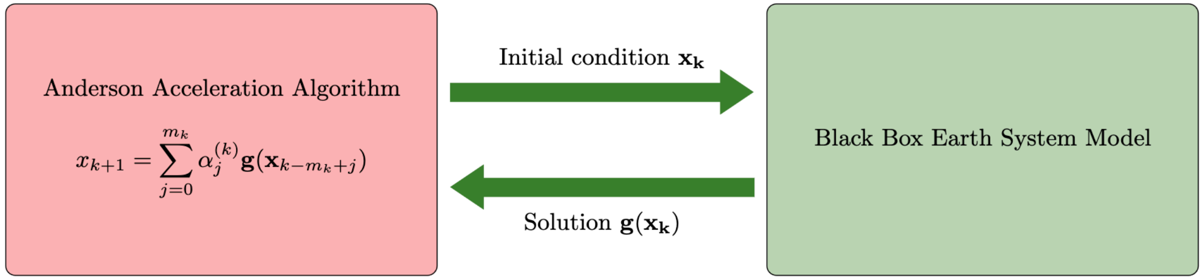
OXFORD EARTHSCIENCES

- We have a sequence of previous iterates: $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots\}$
- Anderson forms linear combination of previous m iterates $$\mathbf{x}_{k+1} = \sum_{i=0}^{m_k} \alpha_i^{(k)} \mathbf{g}(\mathbf{x}_{k-m_k+i})$$
- The coefficients α_i are determined easily through a constrained least squares problem:
  - $$\min_{\alpha_i} \left\| \sum_{i=0}^{m_k} \alpha_i^k \mathbf{f}(\mathbf{x}_{k-m_k+i}) \right\|_2^2$$ subject to $$\sum_{i=0}^{m_k} \alpha_i^k = 1$$
  - Here we are minimizing the residual: $\mathbf{f}(\mathbf{x}) = \mathbf{g}(\mathbf{x}) - \mathbf{x}$
- We have found m=50 works well for ocean models, m=10-15 works well for land

Homer Walker and Peng Ni. “Anderson Acceleration for Fixed-Point Iterations.” In: *SIAM Journal on Numerical Analysis* 49.4 (2011), pp. 1715-1735.

Implementation and Workflow

- AA code is written in Python, sits as a wrapper outside of the model
- Python wrapper handles model run calls
- Utility functions read/write model restart files



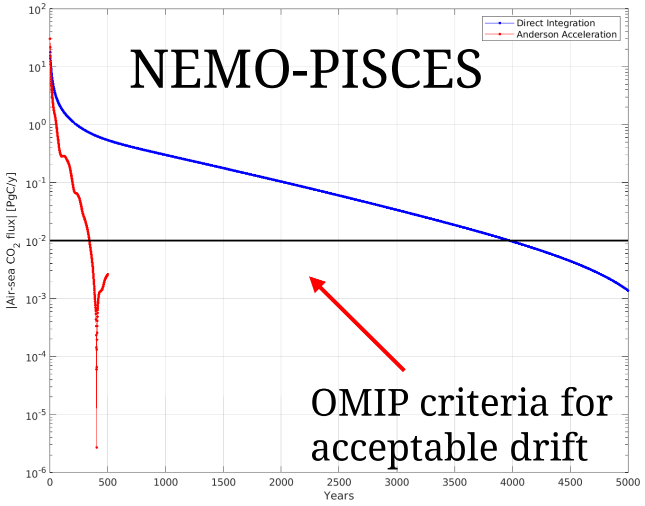
- For ocean models, one instance of AA is run
- For land models, one instance of AA is run ***per soil column***
- Possible to run multiple instances with only a few lines of Python!

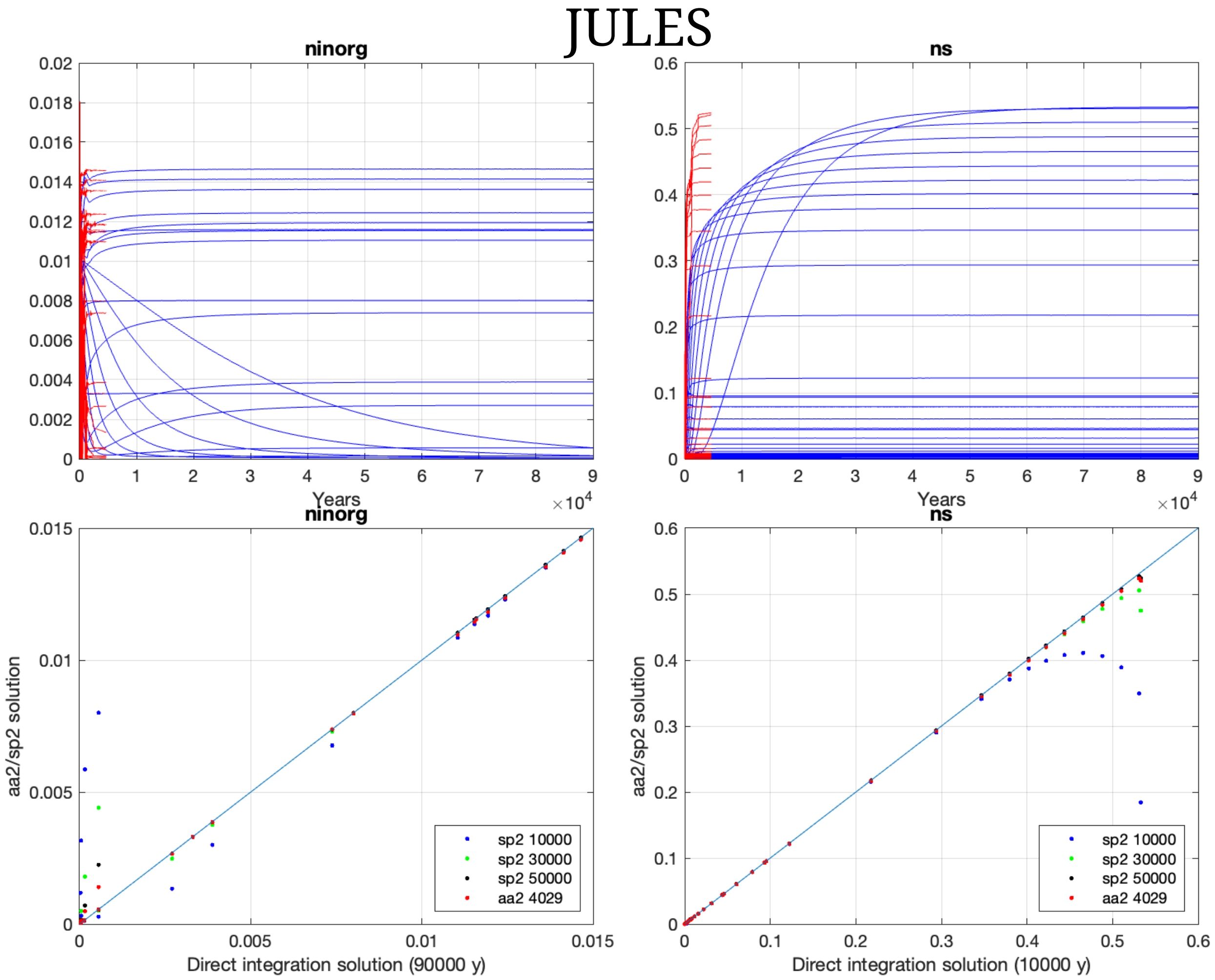
```
finishedJobs=np.zeros(numLand)
while np.any(finishedJobs==0):
  for il in range(numLand):
    suff="_inner_{:06d}.h5".format(il)
    xsol, iter_, ysol, converged = andacc(ginner[il], x0[il], sinner[il], AAparamsInner, histParamsInner[il], 1, suff, y0[il])
    if not np.array_equal(xsol,np.array([])):
      finishedJobs[il]=1
# check again
  if np.any(finishedJobs==0):
    gfunc = lambda x, y, fetchOutput: timestepperfunc_clm(x,y,fetchOutput,dataInner[0],None,1)
    gfunc([],[],0)

  for il in range(numLand):
    x0[il]=sinner[il].x.copy()
    if sinner[il].y is not None:
      y0[il]=sinner[il].y.copy()
```

Past Efforts: Ocean Models and JULES

- We have successfully applied AA to JULES as well as multiple ocean models
- AA is 10-20x faster for JULES, consistently 7-10x faster across NEMO-PISCES, NEMO-MEDUSA, and MITgcm-BLING

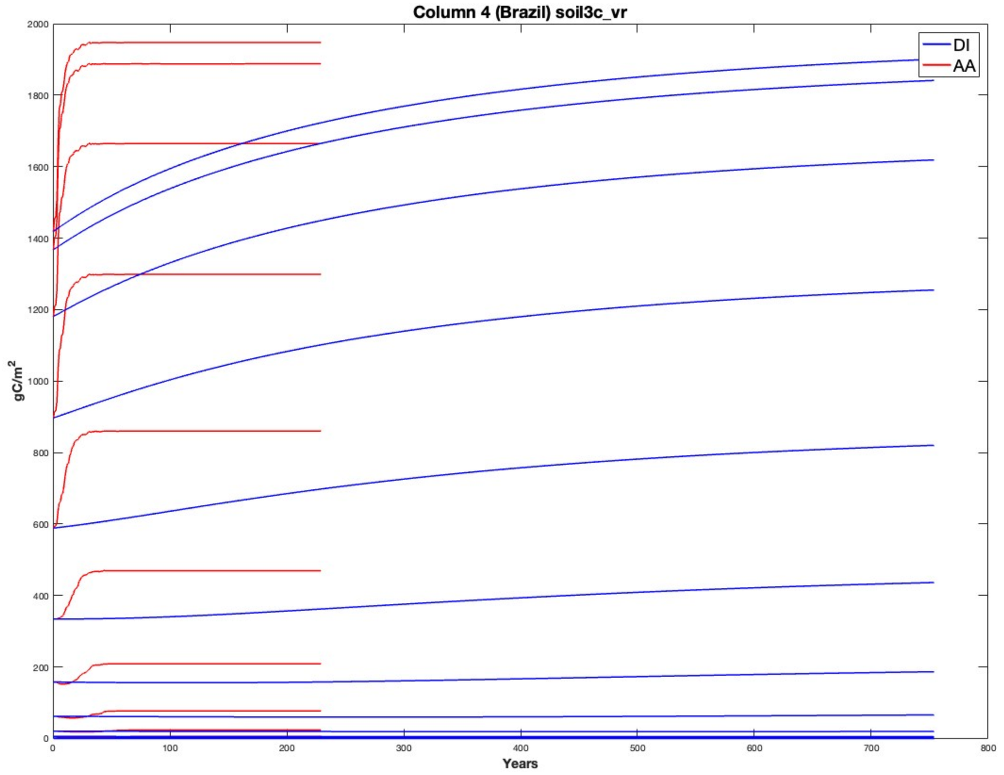


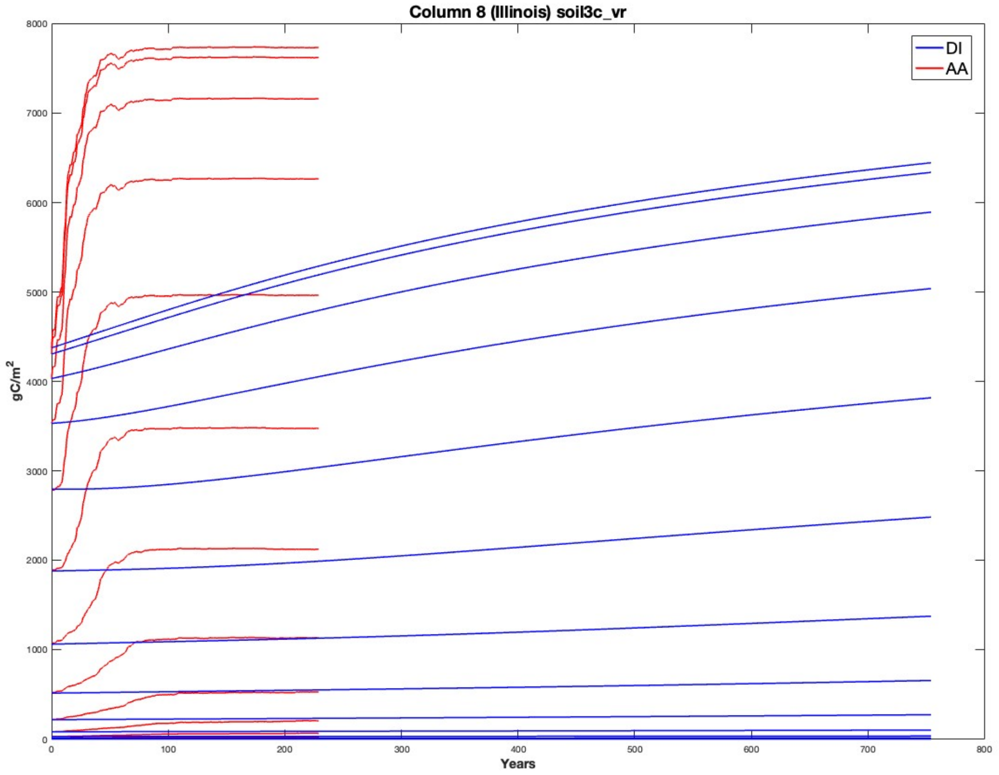


Samar Khatiwala. “Fast Spin-Up of Geochemical Tracers in Ocean Circulation and Climate Models.” In: *Journal of Advances in Modeling Earth Systems* 15.2 (2023).

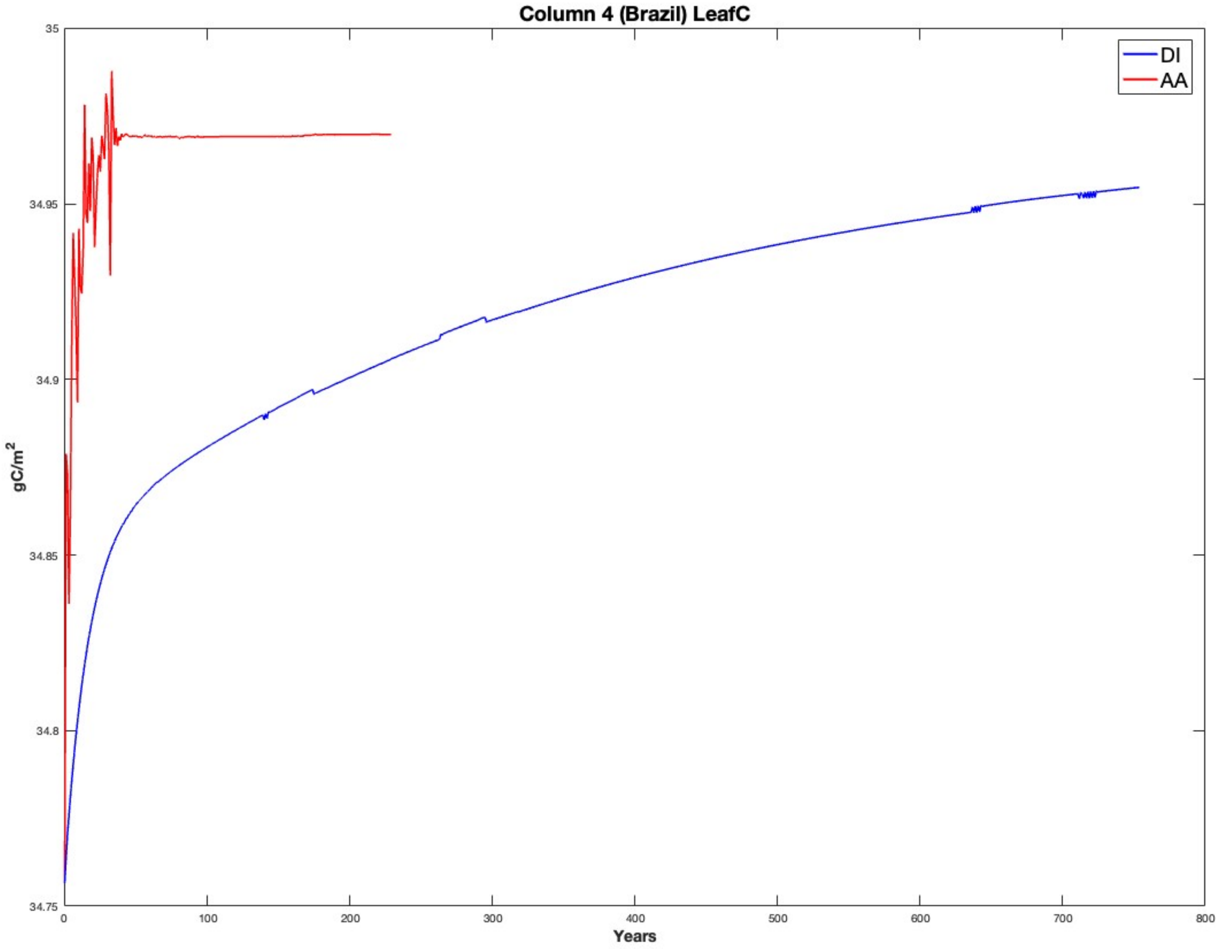
CLM Initial Results: Column Level Tracers

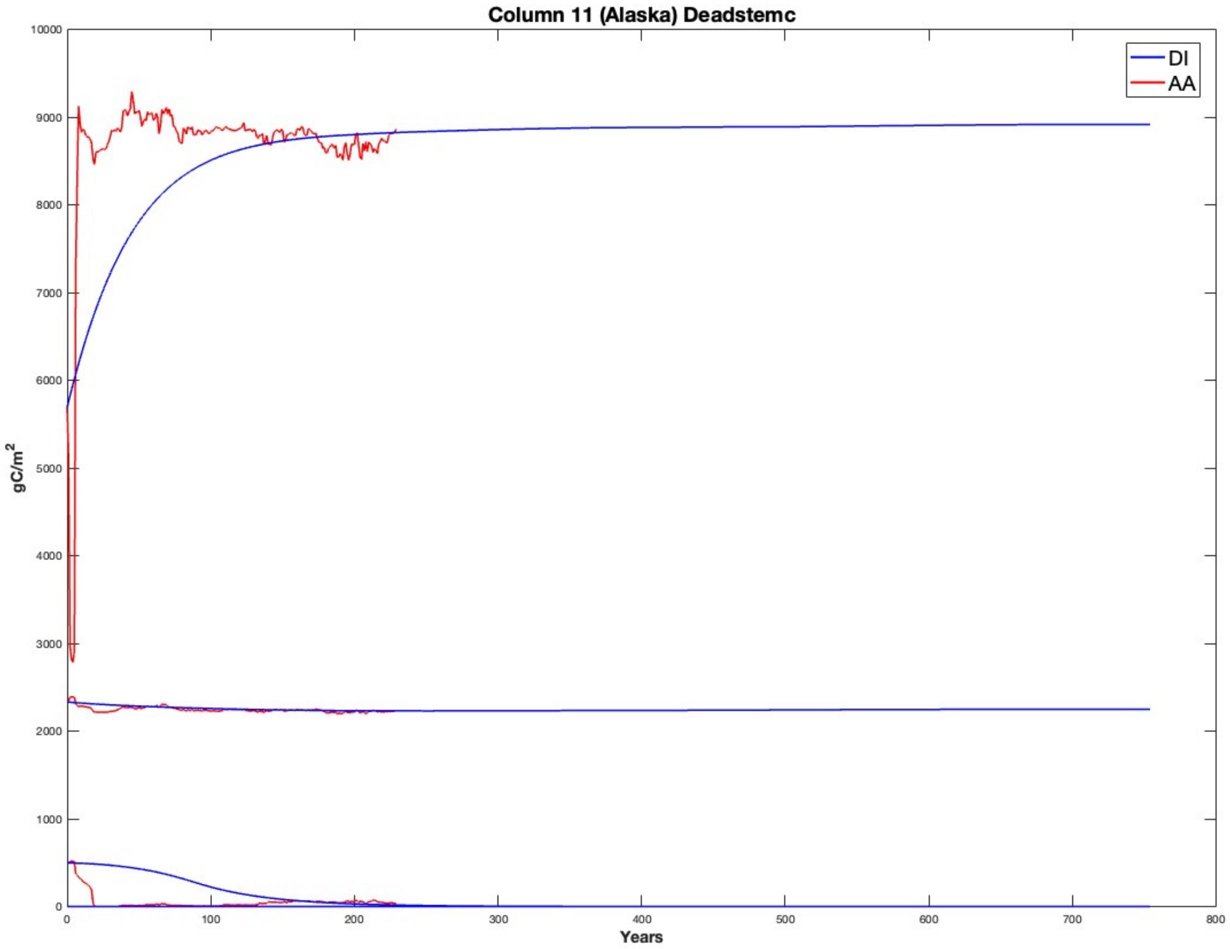
- Initial testing on 3 gridcell setup
  - Points in Alaska, Illinois, Brazil for varied dynamics
- GSWP3v1 forcing, repeating only year 1901, currently C isotopes turned off



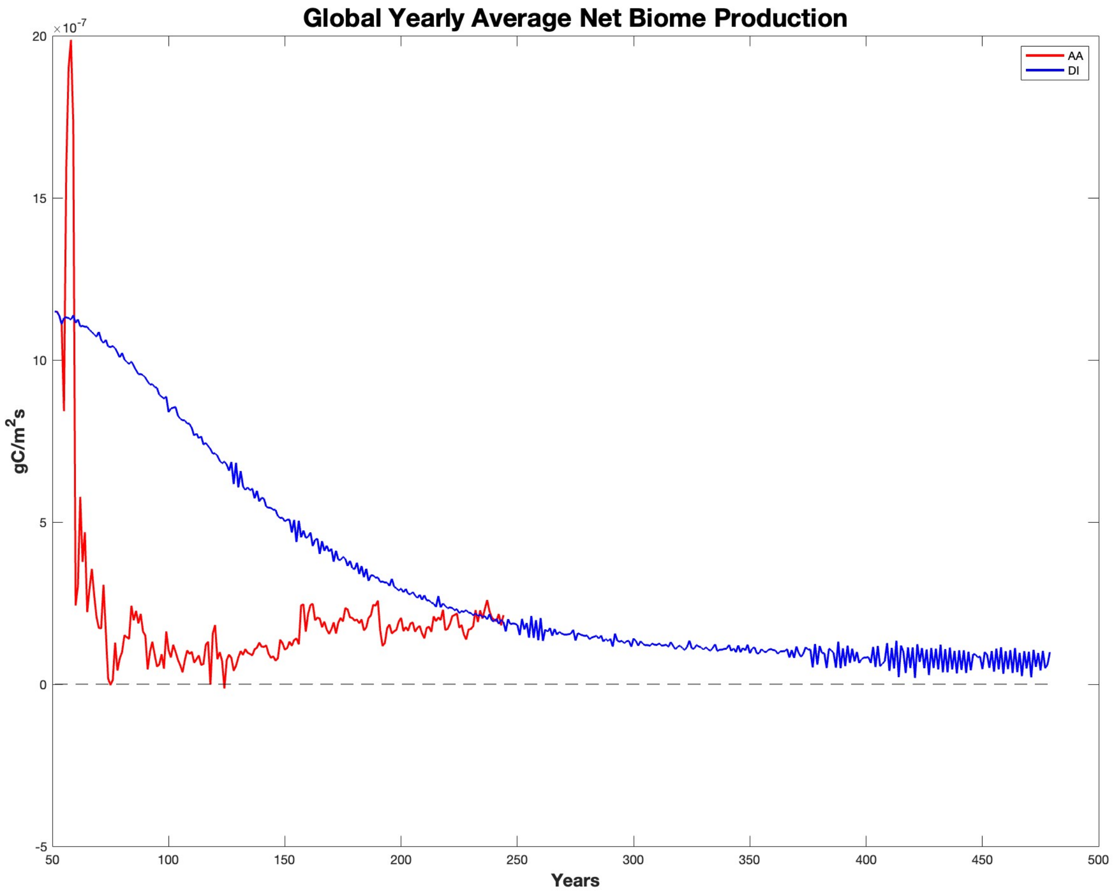


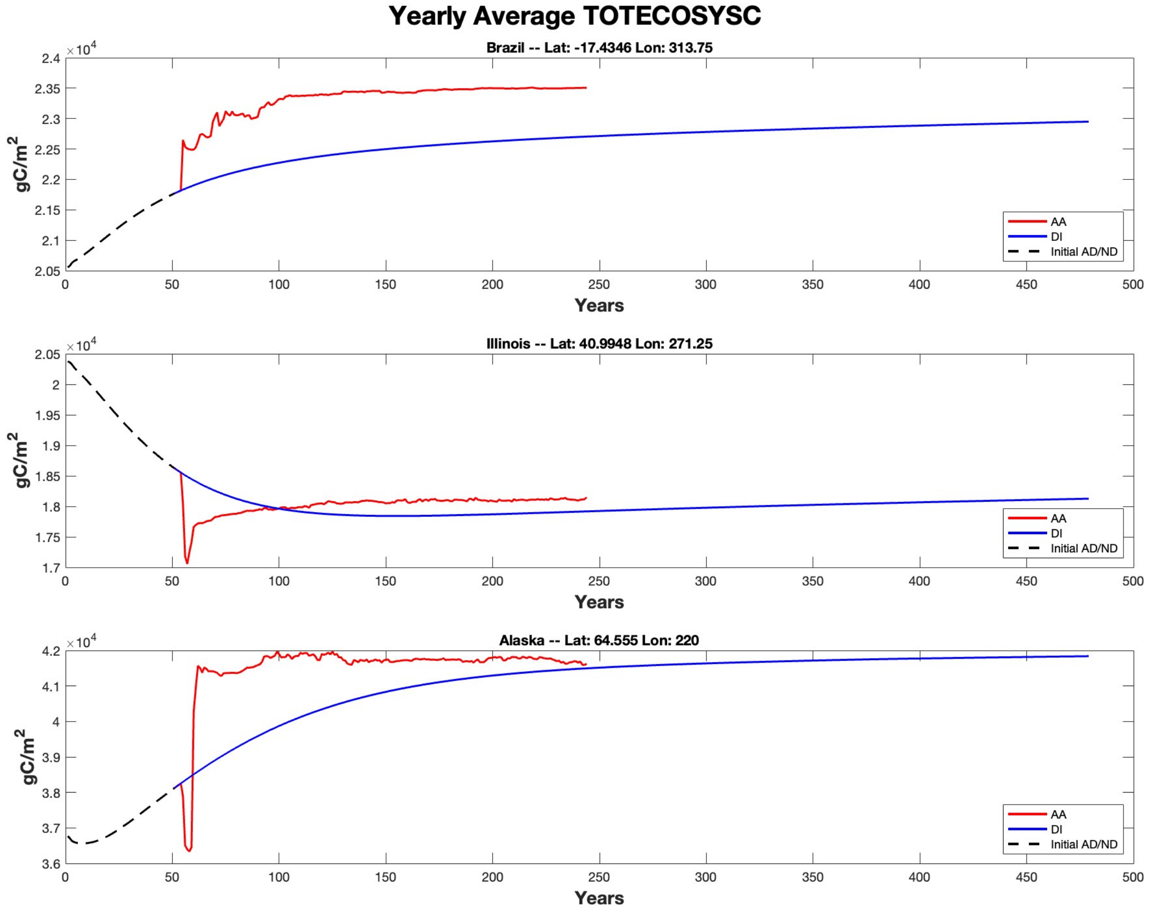
CLM Initial Results: PFT Tracers





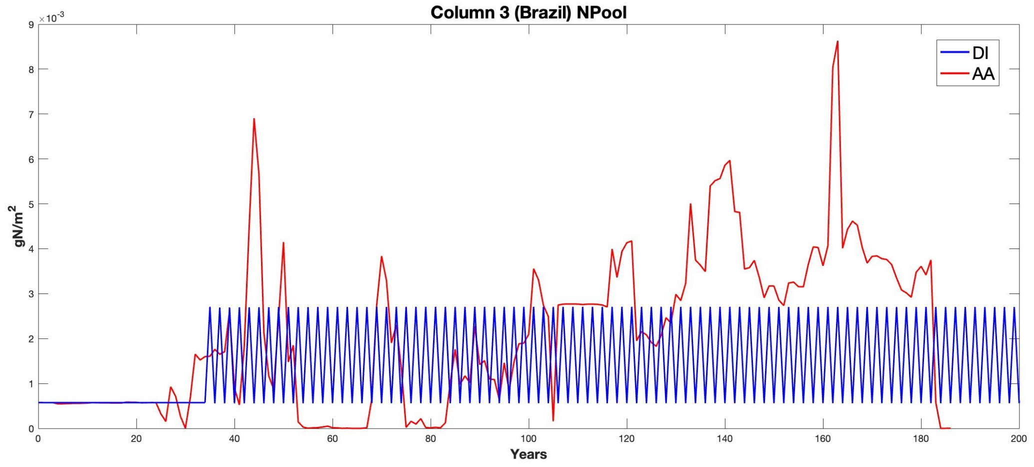
CLM Initial Results: Gridcell and Global Diagnostics

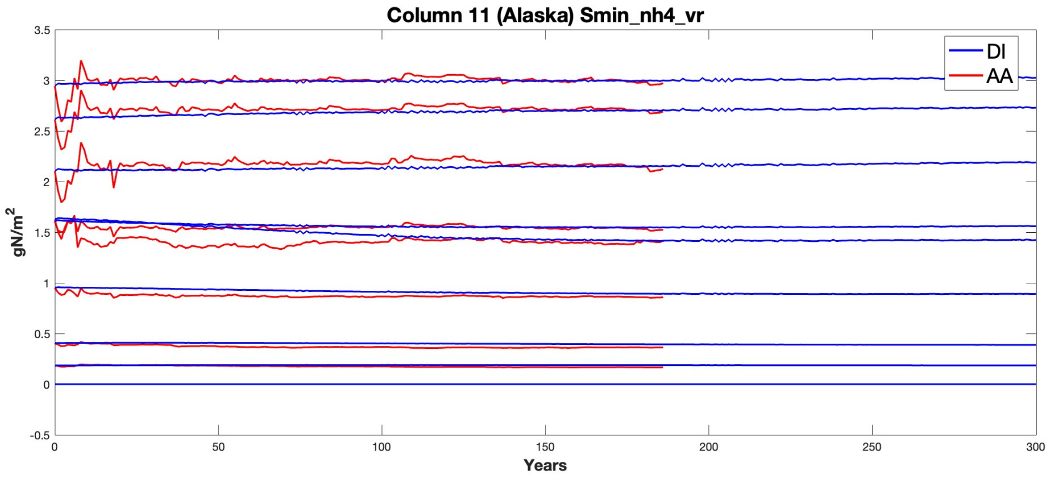


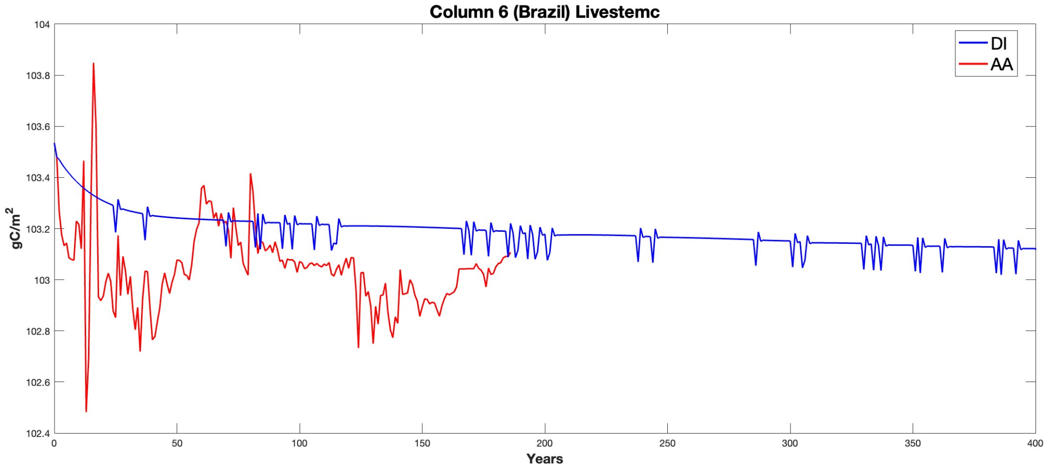


Discussion and Questions

- Model oscillations and noise
- Future directions:
  - Algorithm tuning and convergence criteria
  - Global sparse grid testing
  - Add ^{13}C and ^{14}C isotopes







Further Reading

- Anderson Acceleration for ESMs:
  - Samar Khatiwala. “Efficient spin-up of Earth System Models using sequence acceleration.” In: *Science Advances* 10.18 (2024).
  - Samar Khatiwala. “Fast Spin-Up of Geochemical Tracers in Ocean Circulation and Climate Models.” In: *Journal of Advances in Modeling Earth Systems* 15.2 (2023).
- Mathematics of Anderson Acceleration:
  - Donald Anderson. “Iterative Procedures for Nonlinear Integral Equations.” In: *J. ACM* 12.4 (1965), pp. 547-560.
  - Homer Walker and Peng Ni. “Anderson Acceleration for Fixed-Point Iterations.” In: *SIAM Journal on Numerical Analysis* 49.4 (2011), pp. 1715-1735.
  - Junzi Zhang et al. “Globally Convergent Type-I Anderson Acceleration for Nonsmooth Fixed-Point Iterations.” In: *SIAM J. Optimization* 30.4 (2020), pp. 3170-3197.
  - Haw-ren Fang and Yousef Saad. “Two Classes of Multisecant Methods for Nonlinear Acceleration.” In: *Numerical Linear Algebra with Applications* 16 (2009), pp. 197-221.