CitcomSVE-3.0: An efficient and open-source finite element package for modeling glacial isostatic adjustment process

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Glacial Isostatic Adjustment

Following the deglaciation, the land in formerly glaciated regions (e.g., Hudson Bay above) rebounds, and the seafloor subsides due to the sea level rise that leads to additional loads. This process is called glacial isostatic adjustment (GIA).

Formulation for loading problems (GIA)

- ***Governing equation (compressible medium):***

$$\rho_1^E = -(\rho_0 u_i)_{,i},$$

$$\sigma_{ij,j} + \rho_0 \phi_{,i} - (\rho_0 g u_r)_{,i} - \rho_1^E g_i + \rho_0 V_{a,i} = 0,$$

$$\phi_{,ii} = -4\pi G \rho_1^E,$$

- ***Viscoelastic rheology (e.g., Maxwell body):***

$$\dot{\sigma}_{ij} + \frac{\mu}{\eta}\left(\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}\right) = \lambda \dot{\varepsilon}_{kk}\delta_{ij} + 2\mu\dot{\varepsilon}_{ij},$$

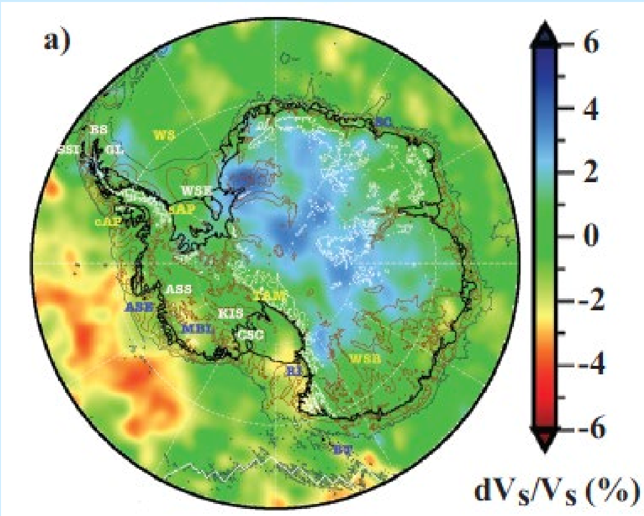
- ***Loads (can also be in terms of potential):***

$$\sigma_{ij} n_j = -\sigma_o n_i, \qquad \text{for } r = r_s,$$

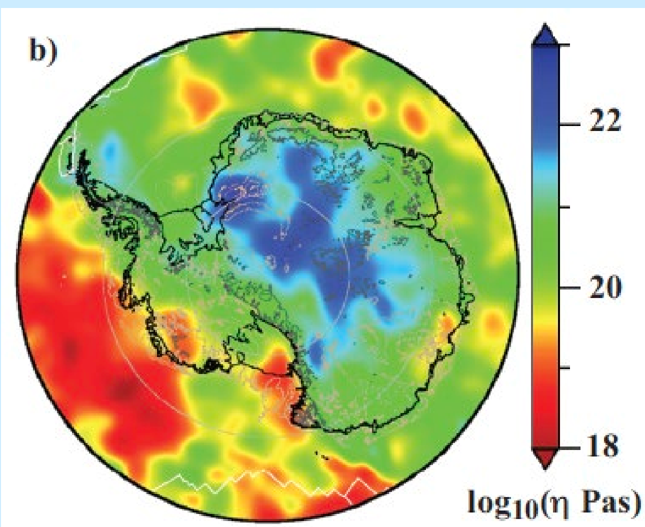
$$\sigma_{ij} n_j = (-\rho_c \phi + \rho_c g u_r) n_i, \qquad \text{for } r = r_b,$$

Inference for 3-D mantle viscosity

300 km depth

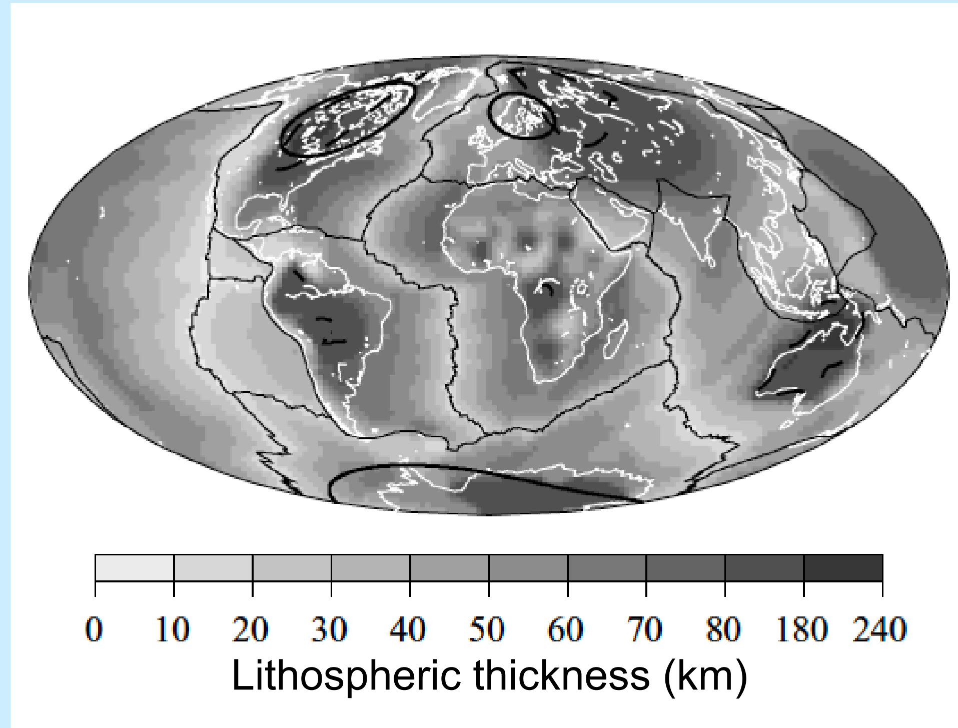


Seismic model ANT-20 [Lloyd et al., 2020]



Ivins et al., [2021, 2022]

Lithospheric thickness variations and weak plate boundaries



Zhong et al. [2003], originally from Watts [2001]

Different realizations of 3D mantle viscosity based on different seismic models and rheological parameters [Bagge et al., 2021]

Viscoelastic rheology (Maxwell and compressible)

Compressible, Maxwell rheology:

$$\dot{\sigma}_{ij} + \frac{\mu}{\eta}\left(\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}\right) = \lambda\dot{\varepsilon}_{kk}\delta_{ij} + 2\mu\dot{\varepsilon}_{ij},$$

Strain partitioning to viscous and elastic parts:

$$\varepsilon_{ij} = \varepsilon_{ij}^{v} + \varepsilon_{ij}^{e} = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \Big/ 2.$$

Discretize the rheology in time as:

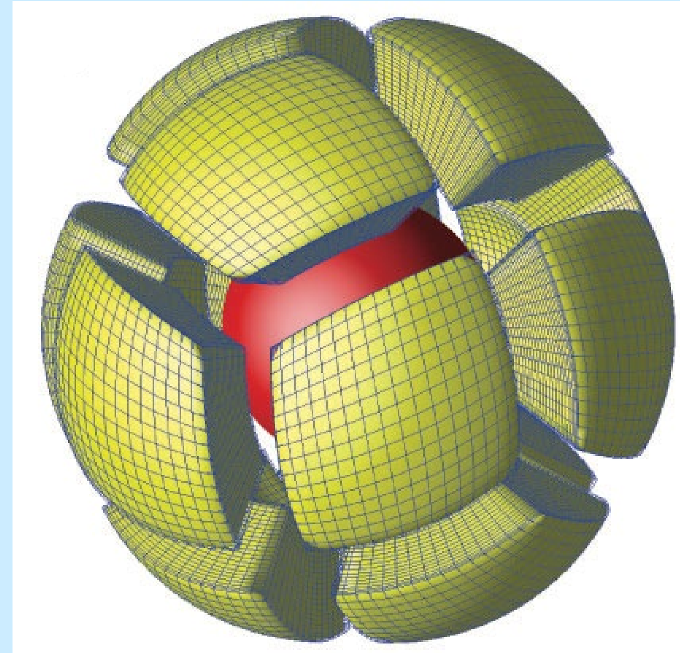
$$\sigma_{ij}^{n} = \tilde{\lambda}\Delta\varepsilon_{kk}^{n}\delta_{ij} + 2\tilde{\mu}\Delta\varepsilon_{ij}^{n} + \tau_{ij}^{pre},$$

Pre-stress

Because of displacement being used as the primary variable, a Lagrangian formulation is often used, where the grid is deformed according to the displacement.

A variational form for finite element analysis (A, Wahr, and Zhong, 2013; Zhong et al., 2003):

$$
\begin{aligned}
&\int_\Omega w_{i,j}[\tilde{\lambda} v_{k,k}\delta_{ij} + \tilde{\mu}(v_{i,j} + v_{j,i})]dV - \int_\Omega \rho_0 g(w_{i,i}v_r + w_r v_{i,i})dV \\
&+\sum_l \int_S w_r \Delta\rho_l g v_r dS_l = \int_\Omega \rho_0 g(w_{i,i}U_r + w_r U_{i,i})dV - \int_\Omega w_{i,i}\rho_0 \phi dV \\
&-\int_\Omega w_{i,j}\tau_{ij}^{pre} dV + \sum_l \int_{S_l} w_r(\Delta\rho_l \phi - \Delta\rho_l g U_r + \rho_0 V_a)dS_l - \int_S w_r \sigma_0 dS
\end{aligned}
$$



CitcomSVE (VE stands for Visco-Elastic) as a parallel code

This leads to a matrix equation for displacement V:

$$[K]\{V\} = \{F_0\} + \{F(\Delta\phi)\},$$

Then use the same solution techniques as for mantle convection codes (e.g., full multi-grid as in CitcomS) to solve this matrix equation.

Development of CitcomSVE and its Applications

1) Adopted from mantle convection code CitcomS [Zhong et al., 2000], CitcomSVE has been used for GIA problems (including degree-1 motion, sea-level equation, true polar wander, compressibility, stress-dependent viscosity) [Zhong et al., 2003; Paulson et al., 2005; A et al., 2013; Kang et al., 2022; Yuan et al., 2024].

2) Regional scale volcano loading with non-linear (frictional, low-temperature plasticity, and power-law) rheology [Zhong & Watts, 2013; Bellas et al., 2020].

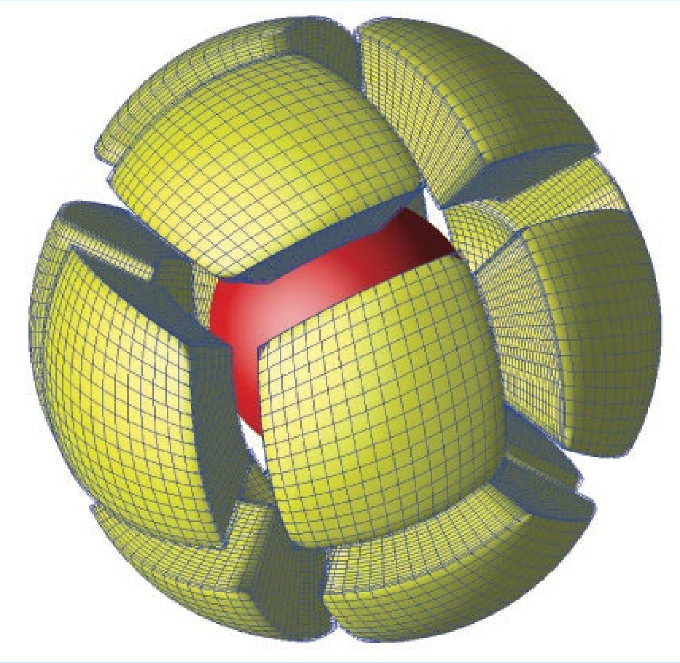
3) Tidal and rotational forcing loading [Fienga et al., 2024; Zhong et al., 2012; Qin et al., 2014, 2018].

N_c[a]	Grid per cap (12x)	CPU time[b] (sec)	Iterations[c] (g/p/v)	CPU time per v-iteration[d] (sec)	Efficiency (%)
12 (1x1x1)	32x32x32	6.45	7/32/56	0.115	100
24 (2x1x1)	64x32x32	10.4	8/36/89	0.117	98.3
48 (1x2x2)	32x64x64	6.00	7/31/44	0.136	84.6
96 (2x2x2)	64x64x64	7.90	7/34/57	0.138	83.3
192 (1x4x4)	32x128x128	8.01	7/33/57	0.141	81.6
384 (2x4x4)	64x128x128	6.67	8/33/44	0.152	75.7
768 (4x4x4)	128x128x128	7.93	8/35/54	0.147	78.2
1536 (2x8x8)	64x256x256	8.87	8/34/61	0.145	79.3
3072 (4x8x8)	128x256x256	20.4	9/74/118	0.173	66.5
6144 (8x8x8)	256x256x256	13.6	12/51/94	0.145	79.3

Zhong et al. [2022]

Key features of CitcomSVE-3.0

- ***A finite element package for computing dynamic deformation of a 3-D spherical shell in response to surface and tidal loads.***
- ***For temperature- and stress-dependent viscosity and 3-D elastic structure, either compressible or incompressible.***
- ***A non-Gauss-Legendre grid with uniform resolution (i.e., no excessive resolution near the poles).***



A non-Gauss-Legendre grid in CitcomSVE that is naturally suited for parallel computing [Zhong et al., 2000; 2003].

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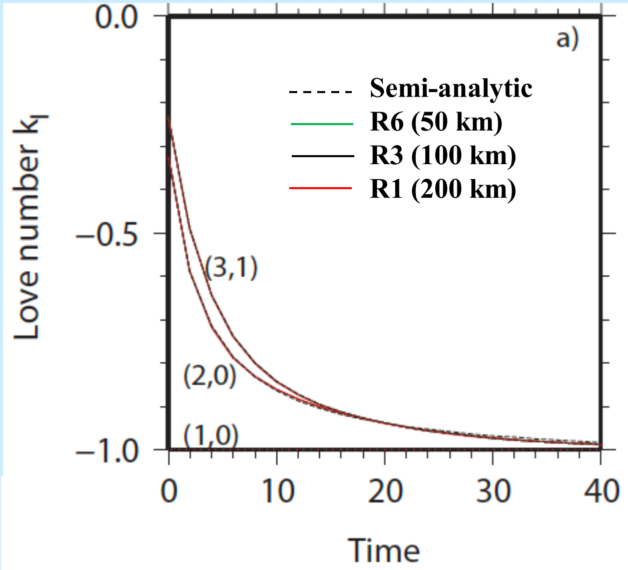
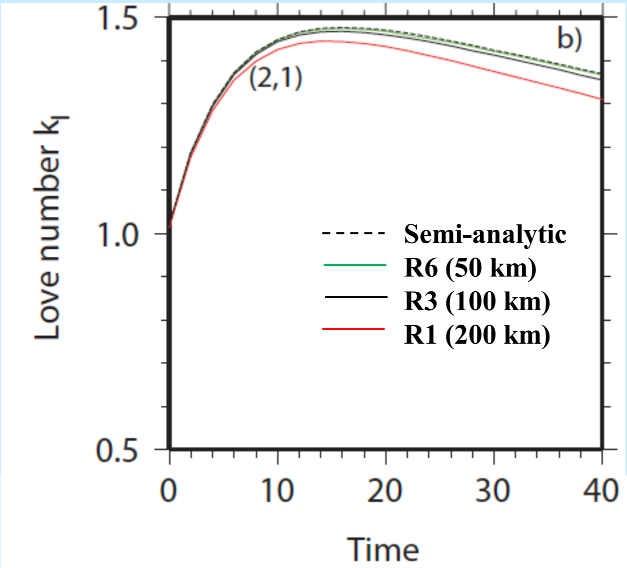
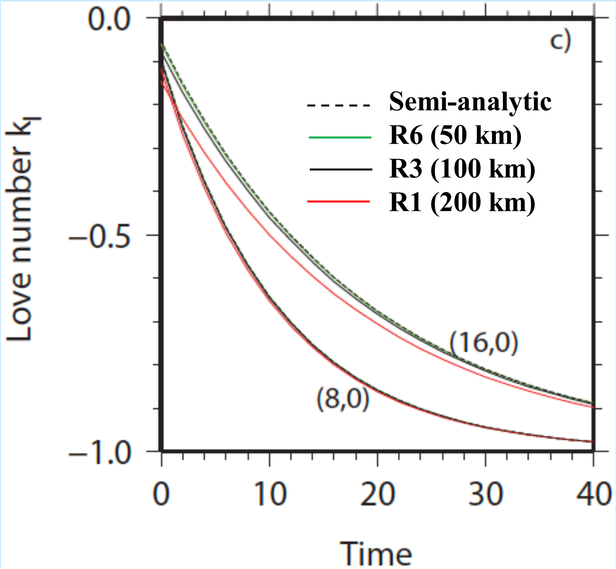
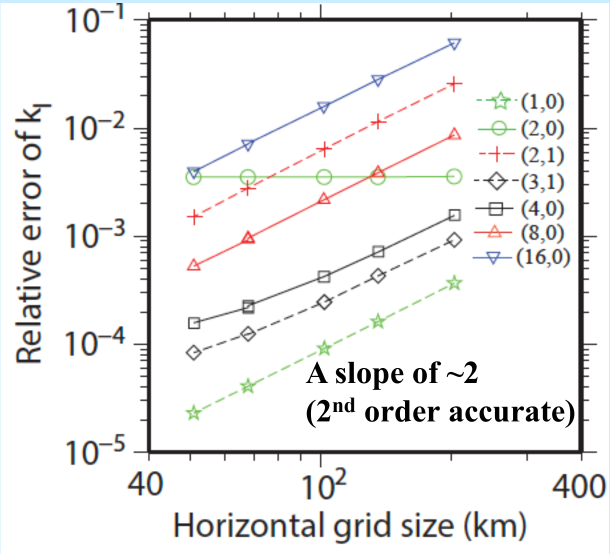
- ***A non-Gauss-Legendre grid with uniform resolution (i.e., no excessive resolution near the poles).***
- ***Efficient on PC-clusters (~100's cores) and massively parallel computers (>6,000 cores).***
- ***Open-source, user's guide, example cases, and publicly available at https://github.com/shjzhong/CitcomSVE***
- ***Planning for online user workshops on Zoom this year (contact me, if interested).***

Benchmark of Load Love numbers h_l, k_l, and l_l



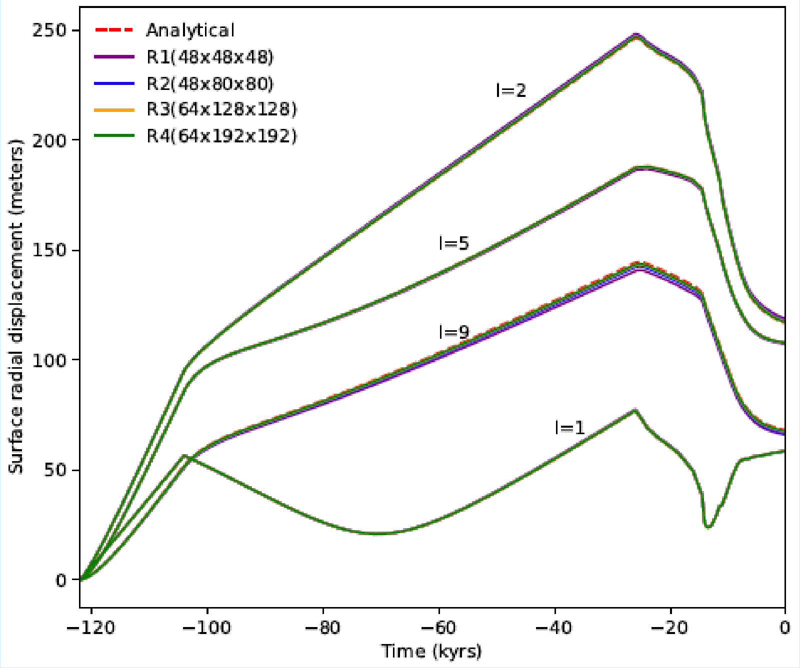


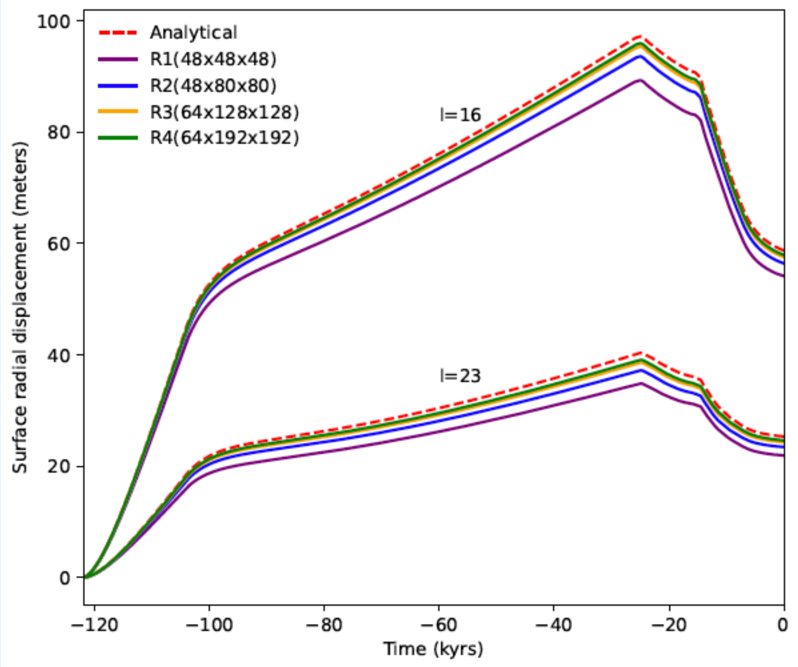




Zhong et al., [2022]

GIA benchmark for ICE-6G and VM5a





Yuan et al., [2024]

It takes ~10 hours computing time using 384 CPU cores on NCAR Supercomputer Derecho to complete a model run of a glacial cycle with ~35 million elements (~30 km horizontal and 10 km vertical resolutions) and ~1000 time steps.

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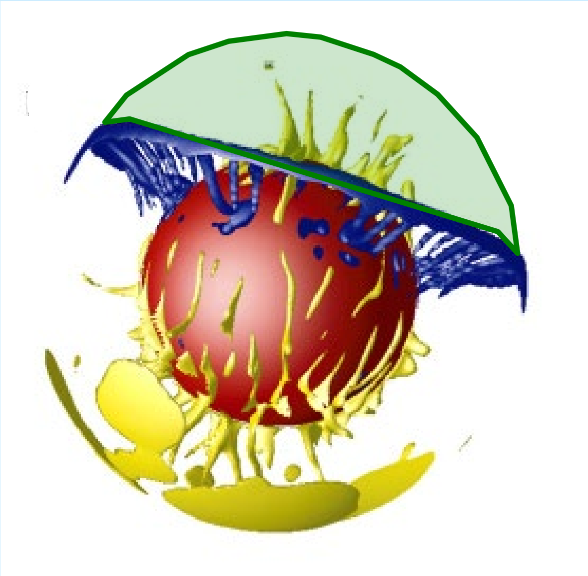
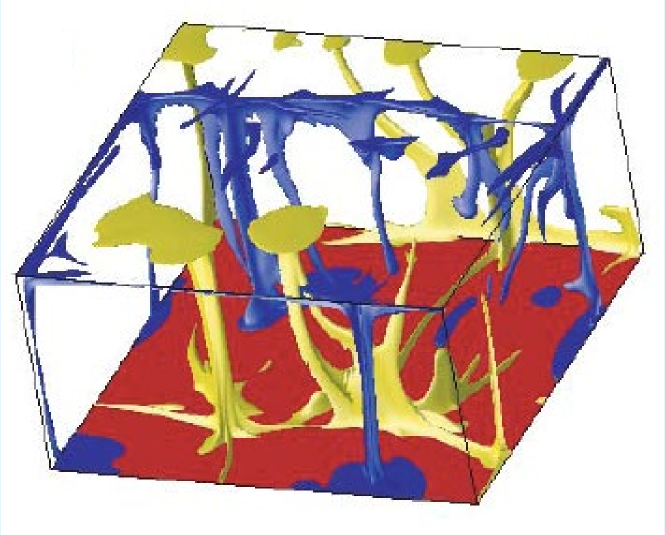
Mantle convection models

Global – CitcomS [e.g., Zhong et al., 2000, 2008]



Regional spherical & Cartesian



1) **Mantle convection models have been well developed and tested over the years.**
2) **Advanced computational technologies (e.g., on parallel computers or clusters with 10s to 10,000s CPUs), numerical algorithms for 3-D, and realistic rheology.**
3) **Publically available (e.g., CitcomS, ASPECT, … are freely downloadable from CIG, well documented and supported).**

Would not it be great if we can use the same technologies for modeling elastic and viscoelatic loading problems?