A Weighted Essential Non-oscillatory Tracer Advection Scheme in MOM6

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Overview

Implementation of high-order WENO-based solver for tracer in MOM6

Evaluation of the WENO-based solver against the PPM solver

Motivation for High-Order Schemes

- Low-order schemes often suffer from numerical dissipation.
- Higher-order methods provide better accuracy and are less dissipative.
- Need for high accuracy while preserving sharp gradients and avoiding numerical oscillations.
- WENO reconstruction achieves high accuracy while effectively handling discontinuities without generating spurious oscillations.

Wang et al. (2023), Silvestri et al. (2024)

Tracers in MOM6

- Passive tracers
  - No effect on momentum or the equation of state
  - Examples: biogeochemistry, transient tracers, or dye tracer

- Active tracers
  - Affect momentum equations through the equation of state
  - Examples: temperature and salinity

- MOM6 solves the tracer transport with no distinction between active and passive tracers

Refer to MOM6 documentation for more details: https://mom6.readthedocs.io

Overview of WENO scheme

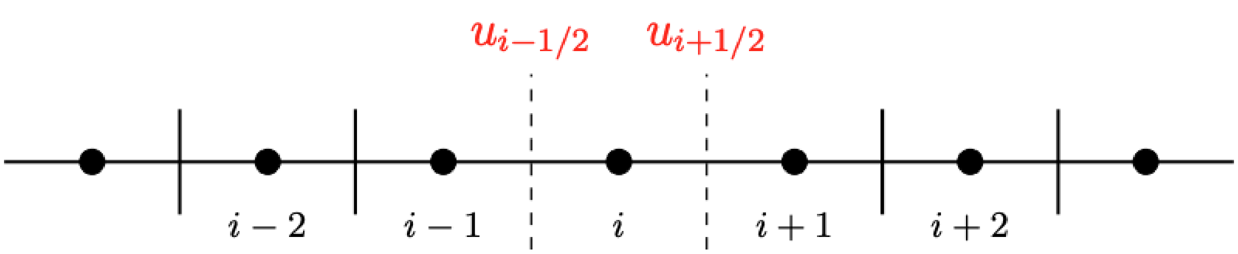
Consider the following one-dimensional linear advection equation

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0,$$

with constant advection speed a. Using a finite volume spatial discretization, we have

$$\bar{u}_i^{n+1} = \bar{u}^n + \nu\left[u_{i+1/2} - u_{i-1/2}\right],$$

where $\nu = a\Delta t/\Delta x$. We use a 5th or 7th-order WENO scheme to compute the fluxes $u_{i+1/2}$ and $u_{i-1/2}$.



Finite Volume WENO5 Stencil for Flux Reconstruction

Reference : Borges et al. (2008); Balsara et al. (2016)

Overview of WENO scheme

- WENO dynamically approximates a numerical flux using multiple low-order reconstructing polynomials.
- These polynomials are combined using nonlinear weights based on smoothness indicators.
- Smoother polynomials receive higher weights, ensuring accuracy in smooth regions.
- Achieve high-order accuracy in smooth regions and lower-order accuracy near discontinuities.
- WENO is total variation bounded, which implies no Gibbs phenomena but can have ripples near fronts.

Wang et al. (2023), Silvestri et al. (2024), shu (2020)

Monotonicity-Preserving WENO scheme

The standard WENO is not monotonicity-preserving (MP), e.g. salt could become negative

Goal: bound the numerical flux $u_{i+1/2}$ so that its value is monotonicity preserving (MP)

- The initial flux $u_{i+1/2}$ is maintained or modified in the MP method
- Find u^{min} and u^{max}, the minimum and maximum bounds within which the flux the $u_{i+1/2}$ lies

$$
\begin{aligned}
u^{min} &= \max\left(\min(\bar{u}_i, \bar{u}_{i+1}, u^{MD}), \min(\bar{u}_i, u^{UL}, u^{LC})\right) \\
u^{max} &= \min\left(\max(\bar{u}_i, \bar{u}_{i+1}, u^{MD}), \max(\bar{u}_i, u^{UL}, u^{LC})\right)
\end{aligned}
$$

  The values of u^{MD}, u^{UL}, u^{LC} are defined in Suresh & Huynh (1997)

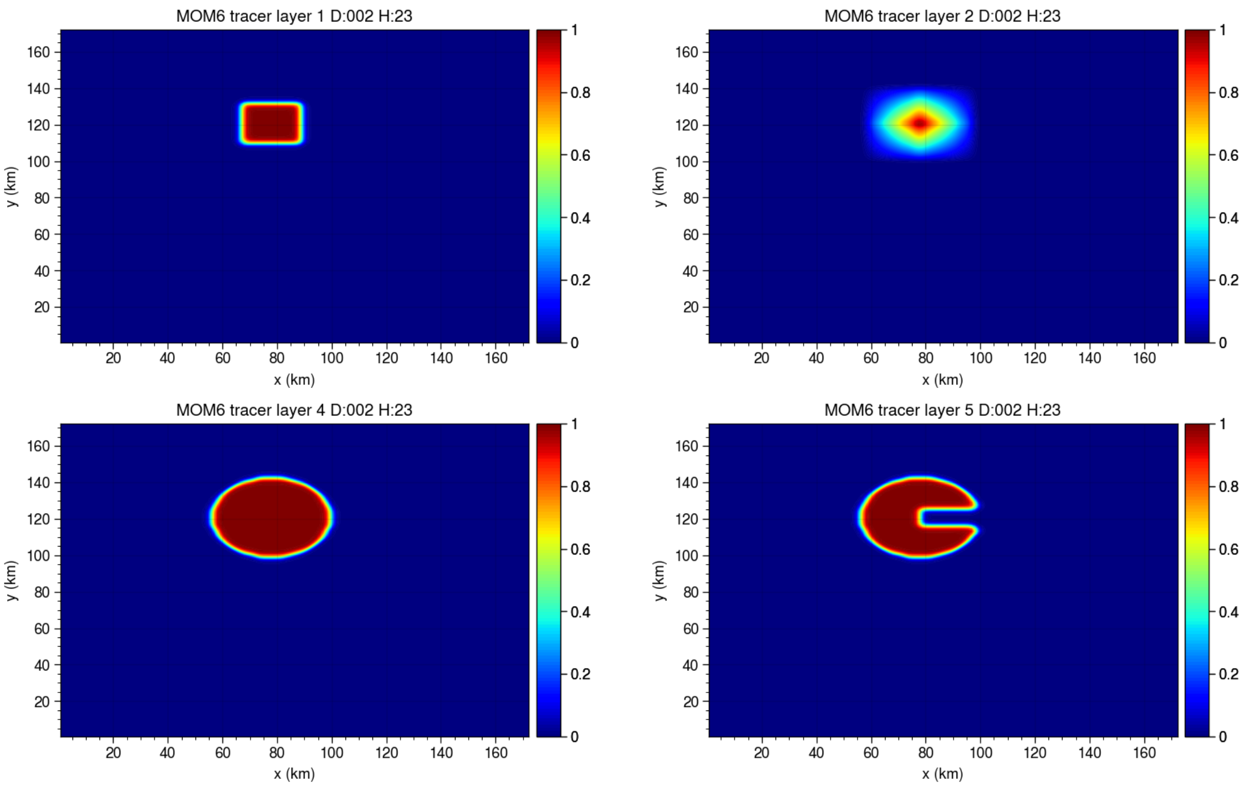
- WENO flux reconstruction becomes

$$
\tilde{u}_{i+1/2} = u_{i+1/2} + \text{minmod}(u^{min} - u_{i+1/2}, u^{max} - u_{i+1/2})
$$

WENO-based solver verification

Advection equation: $$\frac{\partial u}{\partial t} + \nabla \cdot u = 0, \ (x, \ y) \in [0, \ 172.8] \times [0, \ 172.8] \ \text{km}^2$$

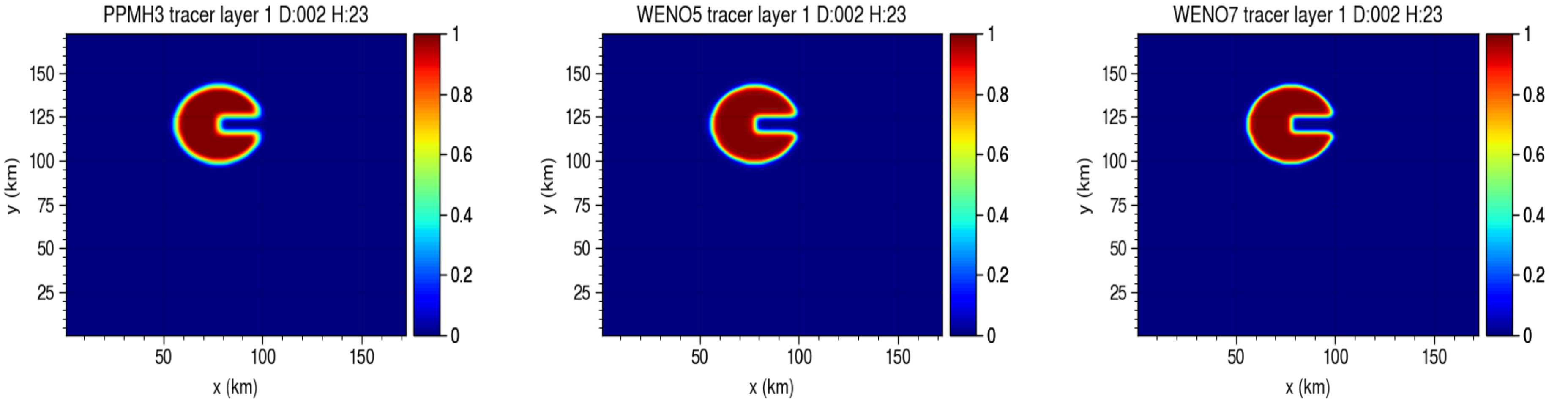
Domain: 172.8 km x 172.8 km using 1.2 km grid resolution



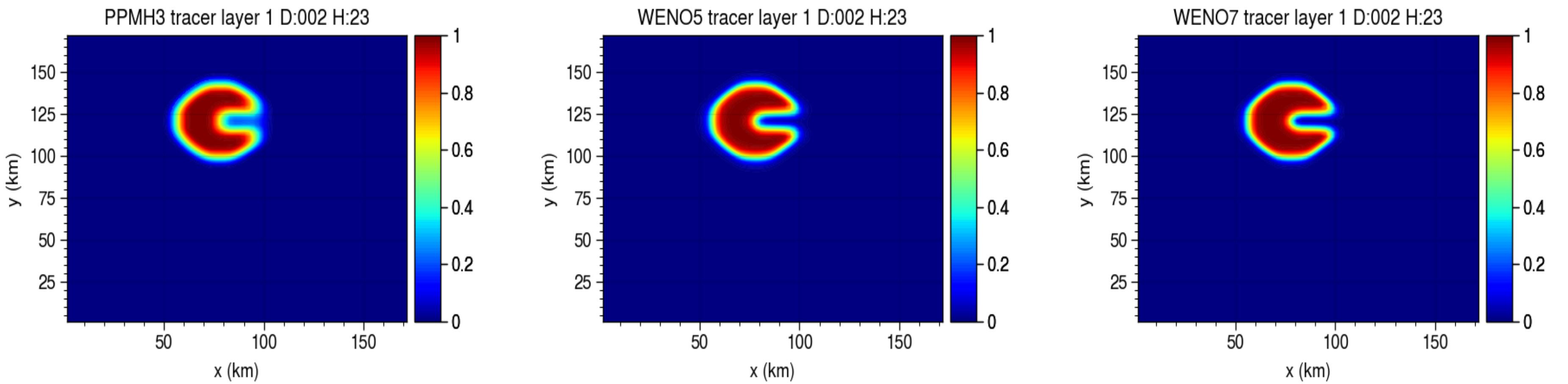
- The solver is stable
- Less dissipative

Comparison WENO and PPM:H3 schemes

Domain: 172.8 km x 172.8 km using 1.2 km grid resolution

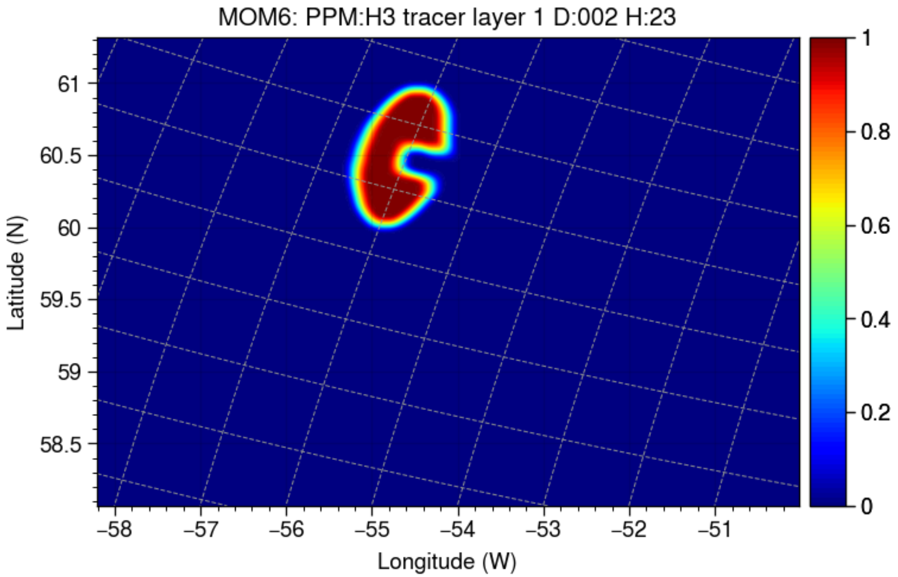


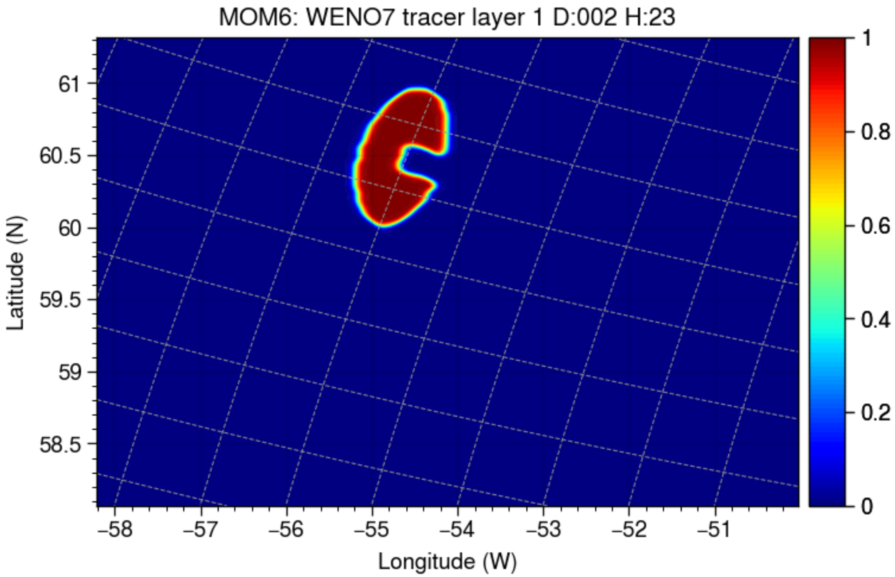
Domain: 172.8 km x 172.8 km using 2.4 km grid resolution



Evaluation of WENO on curvilinear grid

Domain: 58^0 to 50^0 W and 58^0 to 61.5^0 N with 201 x 201 grid points



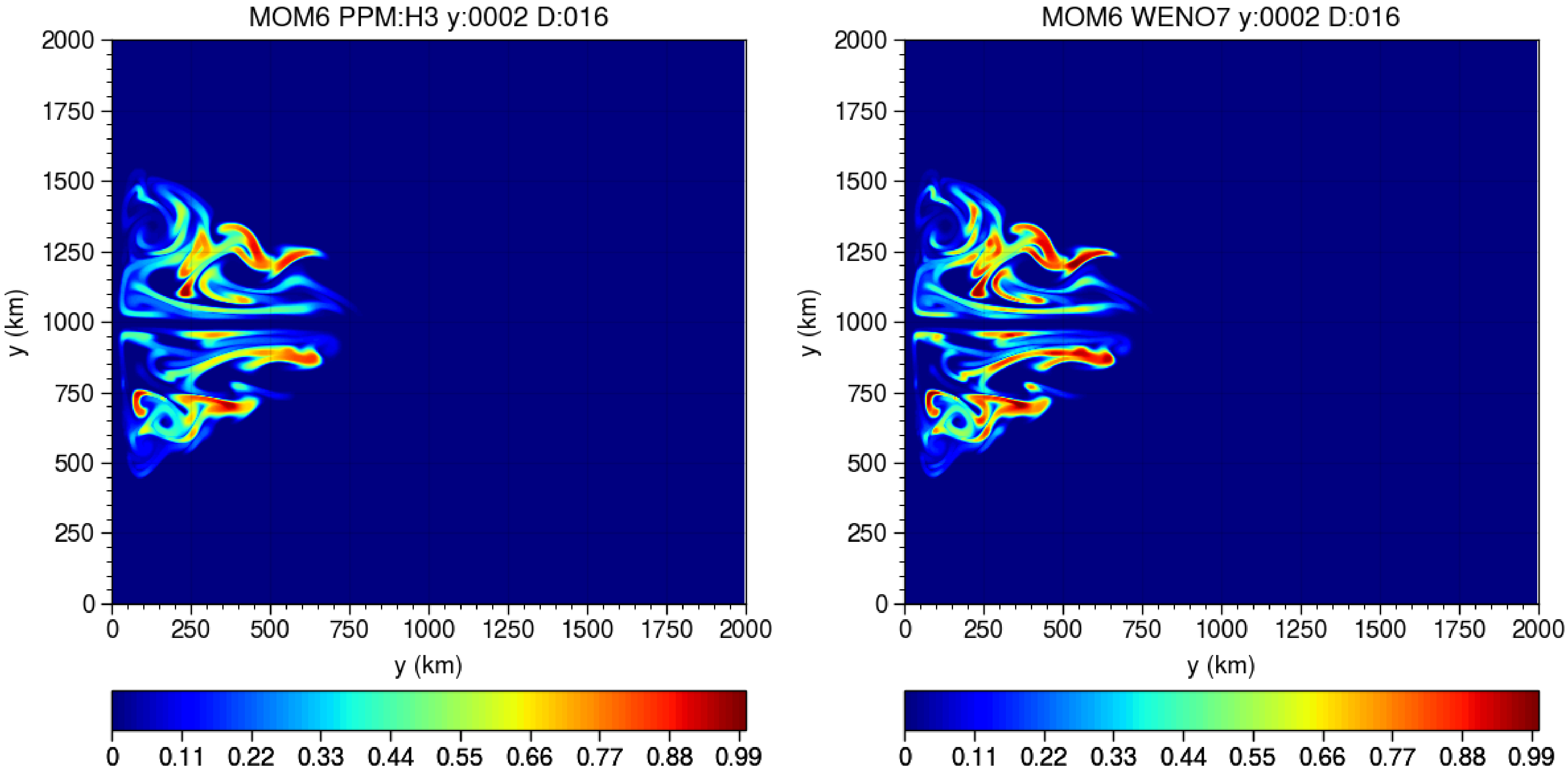


Double-gyre test with passive tracer

To validate the WENO-based solver, we prescribed a passive tracer in the top layer at the western boundary of the computational domain.

- Domain: 2000 km x 2000 km
- Grid resolution: 4km
- Number of layers: 2
- Tracer is prescribed in layer 1
- Simulation period: 5 years

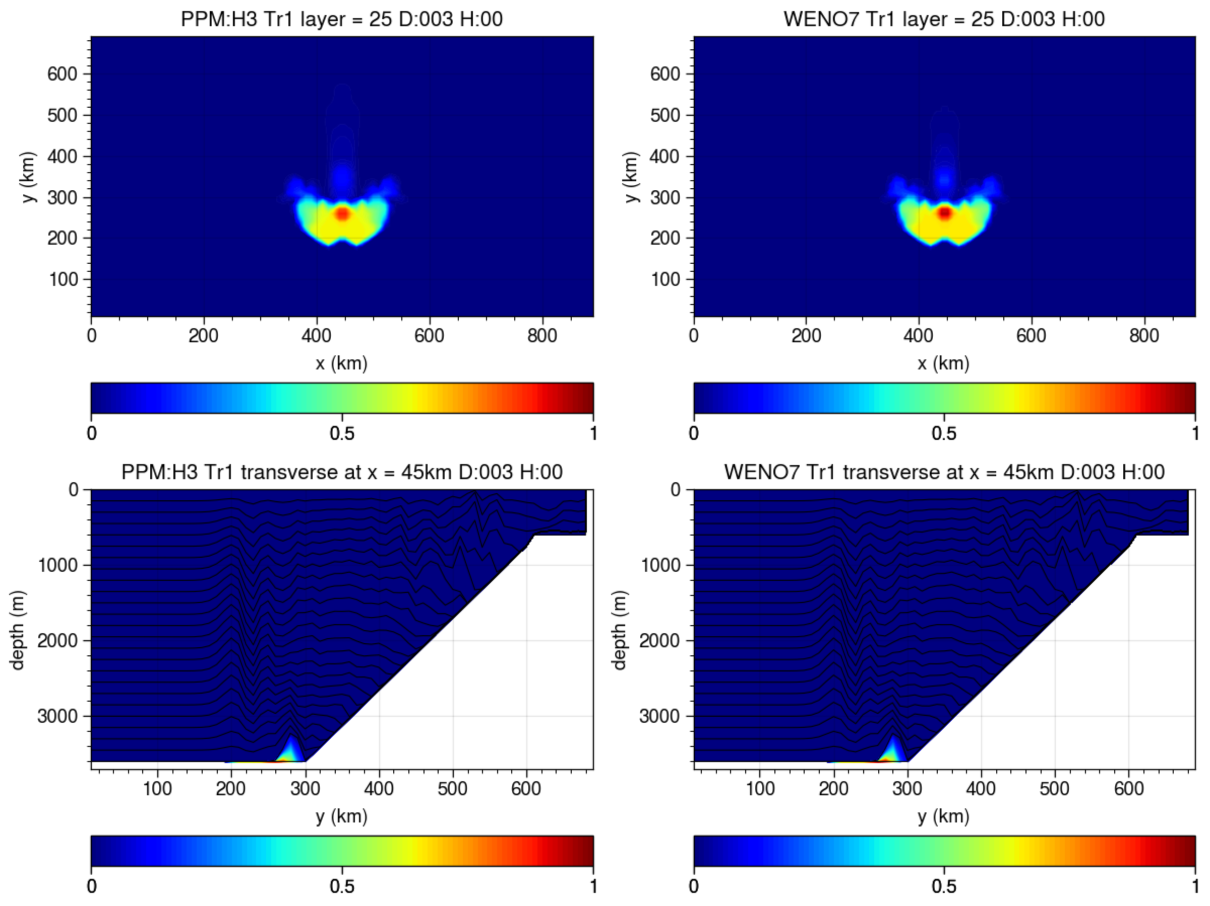
Double-gyre with tracer: PPM:H3 vs WENO7

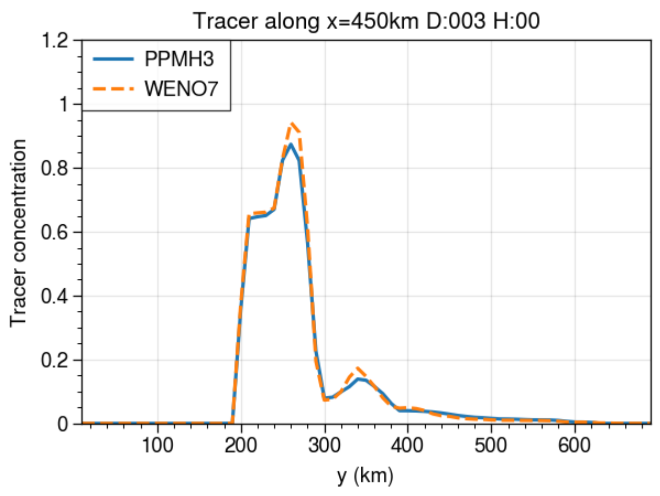


Dambreak test

- 90x70 closed domain, grid resolution 10km, no wind stress
- 25 layers isopycnal
- Bathymetry: in y direction varies from 485m to 3600m
- Drag=2e-4 over 10m.
- CORIOLIS_SCHEME = "SADOURNY75_ENERGY”
- KH_ETA_VEL_SCALE = 0.01
- Boundary conditions: Free-slip

Comparison between PPM:H3 and WENO7

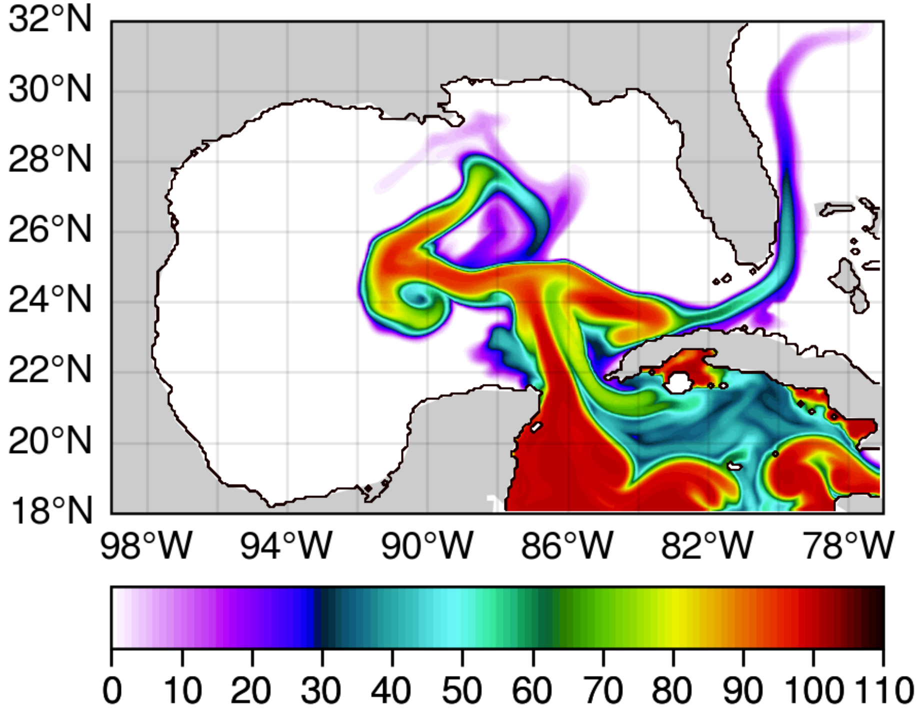




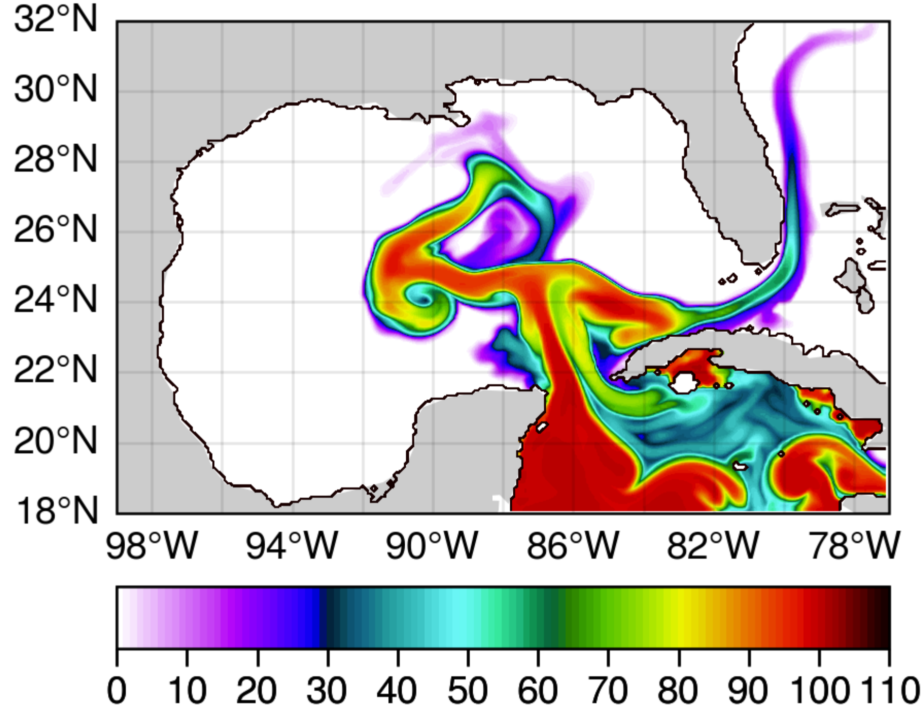
Gulf of Mexico: simulation

- Resolution: 1/12 degree
- Layers: 41
- Duration: 1 year
- Tracer solvers: PPM:H3 vs WENO7

PPM:H3: tracer concentration at 2015-01-31



WENO7: tracer concentration at 2015-01-31



Summary

- We have successfully validated the WENO-based transport solver on:
  - Idealized experiments
  - Realistic experiments
- We observed an improvement in model results
- The scheme is stable and reduces numerical dissipation
- Solvers will be submitted to NOAA-GFDL/MOM6 after further testing and code clean up